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SOME RECOLLECTIONS OF MISS AGNES M. CLERKE.

T. J. J. SEE.

FOR POPULAR ASTRONOMY.

A few months ago the scientific world learned with deep regret of the unexpected death of Miss Clerke, author of the classic history of astronomy during the ninteenth century, and other popular works of distinguished merit. Miss Clerke had reached the age of about sixty-four, but she remained young and hopeful in spirit, and was actively occupied with scientific work up to the very end, which came somewhat suddenly from an attack of grippe passing into pneumonia.

Mr. Hector Macpherson, in the March number of *Popular Astronomy*, and several writers* in England have published notices of the chief events of Miss Clerke's career, in which the details of a busy life are duly chronicled. But there remains something to be said by Miss Clerke's friends in America, who are very numerous. American astronomers not only feel that Miss Clerke was eminently fair as a historian, but many of them, who had the honor of her friendship, felt a great personal loss in her unexpected death, at a time of life when all hoped that many more years of usefulness awaited her.

In a personal letter written shortly after the sad news had reached America, Professor E. E. Barnard of the Yerkes Observ-

* Since the above was written, and forwarded to the Editor, the *Astrophysical Journal* for April 1907 has come to hand, with a very full and very appreciative account of the Life and Work of Miss Clerke, by Lady Huggins, who of all persons in the world is best qualified to speak with authority and with the most intimate knowledge. Every American friend of Miss Clerke will rejoice that so appropriate an account has appeared, and the reader is therefore referred to Lady Huggins' paper, which is much the best available account of Miss Clerke's life yet published. Miss Clerke was the author of several hundred articles, including those on astronomers for the *Dictionary of National Biography*, and the accounts of Humboldt, Laplace, Lagrange and others in the *Encyclopedia Britannica*, besides the following larger works: *History of Astronomy during the Nineteenth Century*, (four editions); *The System of the Stars*, (two editions); *Familiar Studies in Homer*; *The Herschels and Modern Astronomy*; *Concise History of Astronomy*; *Modern Cosmogonies*; *Problems in Astrophysics*.

atory expressed himself as follows: "I am glad you are to write an account of the life and work of Miss Clerke. I agree with you as to her great fairness to observers on this side of the ocean. It was with inexpressible regret that I learned of her death. It is a very great loss to Astronomy, for she has a wonderful insight into astronomical problems and an inexhaustible industry. For my own part, I feel the loss personally very greatly, for in her I lost a true friend. It is a pleasure to remember her kindly greeting to me when I first visited England. At that time her dear mother and sister were alive—but all three have now passed away. There was a heartiness and a kindness about the entire family—including the brother—which I have seldom met with elsewhere."

There are dozens of American astronomers who will cordially concur in Professor Barnard's sentiments; and among these none joins more heartily than the present writer, who also shared the honor of the genial hospitality of Miss Clerke's home while her mother and sister were alive. When traveling in England in the spring of 1892, it was my great good fortune to meet Mr. A. C. Raynard, whose too early death was another great loss to science; and together we saw something of the quiet life and hearty greeting extended to the visiting astronomer at Miss Clerke's home. The impression conveyed by Professor Barnard is a very just one, and probably there are few American astronomers who have been abroad who have not equally precious recollections. One remembers especially her great simplicity of manner, and simple devotion to truth, to which her whole life was consecrated.

She was an omniverous reader, and always made copious notes, so as to have ample references at hand. These references are much valued by the busy astronomer who consults her history and other works, and they insure to these books a lasting value, which seldom is accorded to popular books.

It is sometimes thought that popular works are of less value to the world than the more technical investigations, but one may well doubt whether this is really true. The popular work helps every investigator, as well as the general reader, while the special investigations appeal to very few, and thus make but little impression upon the world's thought. Good popular writing is therefore as necessary to scientific progress as the most technical researches, which can exert but little influence because of their very abstruseness.

In the line of popular writing Miss Clerke had no superior.

Her style was classic, her imagery clear, her thought lucid, original and suggestive. Many an investigation has been suggested to other minds by the reading of Miss Clerke's summary of the results already attained. Thus her influence upon modern thought has been extensive, especially in observational and astrophysical lines. Often by condensing the essential and omitting the non-essential elements of memoirs she has harmonized results apparently contradictory. This is always an important service to science, and most necessary for solid progress.

As a recreation from her more purely scientific studies Miss Clerke occupied herself with the greatest of epic poets, and published a volume of "Familiar Studies in Homer," which has few superiors in a class of literature too little read these days. Her classic inspiration and spirit was drawn from the fountain source of the best literature in all ages.

By some writers regret has been expressed that Miss Clerke was not a laboratory worker. I fail to see the propriety of this implied criticism of her training. Even now the equipment of our observatories is constantly changing, and opinions or methods once in use seldom hold for ten or at most twenty years.

Miss Clerke certainly did not lack in grasp of the essential; on the contrary her grasp was often better than that of many eminent men whose studies have necessarily been restricted to a narrower field. As her work was that of a historian and generalizer, so far as generalization was possible, it is clear that she was pre-eminently qualified for her appointed task. And her success in all the varied labors attempted brought her honors and conspicuous recognition, and a wide circle of sympathetic friends.

Among the women of all ages who have devoted their lives to Science, she easily occupies a foremost place. On her brother's and on her own account Miss Caroline Herschel acquired a lasting fame, and in like manner the achievements of Mrs. Somnerville are held to be remarkable in many ways. But so far as one can now see Miss Clerke has added additional splendor to these illustrious traditions of Great Britain, and will always rank among the most gifted popular writers of any age or country.

One thought more comes to mind. There will now be room for a successor of Miss Clerke, but the intellectual endowment required will have to be as extraordinary as the even balance of the moral powers which render her judgment so discerning. Such persons are not easily found in this age of inferior literature, characterized by haste and immature results; but we may hope

that some one will continue worthily the great work which she has developed. It must be extended and renewed with the lapse of time in order to be a record of the progress of Astronomical Science.

Naval Observatory,
Mare Island, California,
April 15, 1907.

FATHER HAGEN'S ASTROPHOTOGRAPHIC JOURNEY.

WM. F. RIGGE, S. J.

FOR POPULAR ASTRONOMY.

It was in the early part of April, 1906, that Father Hagen took his departure from the Georgetown College Observatory in order to obey the summons of Pope Pius X and assume the directorship of the Vatican Observatory. After making himself acquainted with his new surroundings and taking the inventory of his new possessions, he found that the principal instrument of the observatory was a photographic equatorial of the international type, which Leo XIII had ordered to be made in Paris, and with which the Vatican Observatory had agreed to participate in the construction of the astrophotographic map of the heavens. This share of the work comprised the zone between 55 and 64 degrees of north declination, and entailed the exposure and measurement of 1040 photographic plates. The death of the director, Father Denza, a Barnabite, on December 14, 1894, left Father Lais, an Oratorian, alone with the work. The financial embarrassments of the Vatican added their share to the difficulties of the undertaking, so that as a result only a total of 40 plates has thus far been secured.

The present Pontiff Pius X determined in spite of his own limited means to infuse new life into the observatory. His first act was to call Father Hagen to the directorship and, as much as he was able, to aid him in his work. Seeing therefore that the Vatican had committed itself to the photographic chart, Father Hagen's first care was to find assistants. He retained the services of Father Lais and also secured those of Father John Stein, a Jesuit, from Holland, whose dissertation on the variation of latitude had won him the doctor's degree. His next care was to get ideas concerning the practical execution of the work in hand. For this purpose he made a trip of about two months duration through France, Holland, England and Italy, in order to study

the methods used in the various astrophotographic observatories that are taking part in the work. He has published an account of this journey in the Stimmen aus Maria-Laach, Vol. 72, No. 2. His description and criticisms will, I trust, be of great interest to the readers of POPULAR ASTRONOMY, and therefore form the substance of this article.

But before we follow him on his journey, it may not be amiss to say a few words about the condition of his own Vatican Observatory. I will omit the description of its meteorological part, which may be found in the *Monthly Weather Review* for December 1905 under the title "A Visit to European Observatories."

The first observatory in Rome was erected by Gregory XIII, after whom the Gregorian calendar is named, and built upon the elegant "Tower of the Winds." After suffering various vicissitudes it was awakened to a new and modern life by Leo XIII. By his brief of March 14, 1891 he established the Vatican Observatory upon a secure footing by assigning to it a permanent income. As the Tower of the Winds was not suitable for it, the photographic equatorial was mounted on the Torre Leonina, or Leonine Tower, which Leo IV had built in the year 848 as a bulwark against the incursions of the Saracens, and which therefore acquired a double right to its name. This thousand-year tower is truly colossal in its dimensions, its diameter being seventeen meters and its walls being four meters thick. A wall formerly connected the Leonine Tower with the Tower of the Winds, which are 300 meters apart, but as 85 meters of its length are wanting, archbishop Maffi, the manager of the Vatican Observatory, conceived the idea of restoring this connection by an elegant iron bridge, the cost of which, five thousand dollars, was donated to Father Hagen by an American friend who wishes his name to be withheld.

While Fathers Stein and Lais will attend to the photographic telescope, it is Father Hagen's plan to use Father Müller's ten-inch equatorial for visual purposes. This instrument has already been removed from its unfavorable position at the Gregorian University (Pop. Astr. No. 114, p. 233) and transferred to the Tower of the Winds, now the Torre Pia, the old roof of which has already been removed, but means are wanting as yet for the erection of the telescope and the dome. An adjoining building is designed to contain a photoheliograph, which is at present lying idle in the Vatican Palace. A transit instrument has already been mounted. The present pontiff has made a personal sacrifice to the cause of the observatory by turning into private rooms for

the observers the little summer villa which Leo XIII had built, and in which he used to write his verses and spend the summer nights.

Let us now follow Father Hagen on his journey. After visiting the observatories at Nizza and Marseilles, which were of no special importance for his purpose, he was conducted through the one at Toulouse by the director Baillaud in person. Five ladies attended to the measurement of the plates. The catalogue stars (that is, those between the ninth and eleventh magnitudes, the chart stars being between the ninth and fourteenth magnitudes,) were measured upon three Gauthier instruments. Two others, by the same maker but invented by Baillaud himself, served for the estimation of the magnitudes. The large meridian circle was used for the fundamental stars. The astrographic charts are only photographed here, they are printed by the Paris Observatory. The old 23 cm. equatorial was especially interesting, because it had been changed to a 40 cm. at small cost, the mounting being strong and the dome large enough for the purpose. There was also a reflecting telescope at this observatory.

At Bordeaux the director Rayet had just been buried a week previous. The buildings are far apart and low and built of heavy stones, an advantage that makes them solid and cool. The circular shape of the equatorial towers however is a disadvantage, inasmuch as it leaves little room for tables and chairs. Besides the photographic equatorial of the usual size of 32 cm. there is also a smaller one of 20 cm. As the main building is low, the library had to be distributed among several smaller rooms, but it seemed to be very complete and well arranged. The photographic plates are measured at the University building, and not at the observatory because the distance of the latter from the city would be too great for the ladies. Here there were six ladies at work, each plate being measured only by one person and not reversed 180°. The plates for the astrographic chart are only photographed and developed here, just as at Toulouse, and then turned over to the Paris Observatory.

In Paris the first place to be visited was that of the instrument maker Gauthier, who showed the specifications for his various measuring engines. At the Observatory, as the director Loewy was absent, M. Fraissinet, the secretary, was able to give most complete information in regard to everything connected with the institution, for besides being the secretary of the observatory and conducting its entire enormous correspondence, he is also librarian, receives the large daily mail, opens all packages and

receipts them. He is also treasurer, distributes salaries, pays all bills, and manages the entire business part of the printing and publishing of the photographic chart of the heavens, and this for all the French observatories, that is, for those at Toulouse, Bordeaux and Algiers. M. Fraissinet knows the whole history and every detail of the observatory, since he has held his position under the directors Leverrier, Mouchez, Tisserand and Loewy.

Professor Bigourdan explained his work upon nebulæ. Father Hagen also attended a session of the Academy of Science, the most important of the five academies that constitute the Institut de France, where he met his former acquaintances Poincaré and Darboux, and heard Lippman lecture upon color photography and Janssen upon celestial photography.

The next day he visited Janssen's observatory at Meudon. The large dome, then undergoing repairs, covers the twin telescope, the visual part of which has an aperture of 60cm. There are also three other domes containing a reflector of 1m, a photographic equatorial of about 30cm. and a visual one of about 20cm. aperture. The 15cm. photoheliometer is the sister instrument of the one of the Vatican. It is not mounted equatorially, and is rolled out of its house on rails. A small transit serves only to regulate the clocks. Here in Meudon the large telescope of the Paris Exposition lies packed away. The company that had it built has been able to pay neither the constructor Gauthier nor the exposition place, and imagines that a lottery will restore its lost capital. In this lottery the winner would surely be the loser. The optical part of this giant telescope is not yet completed, as the examination of a star image at the time of the Exposition clearly proved. The tube is laid down horizontally and receives its light from a mirror which floats in mercury. An assistant at the mirror is in telephonic communication with the observer at the eye end. Such a machine might perhaps be useful in a public park.

At the headquarters of the astrographic charts in the Pavillion Dr. Puiseux, the director was absent, but Dr. A. Smeets explained in detail the manner of handling the plates and correcting the proof sheets. Dr. Le Morvan explained how the plates were enlarged for the printing of the charts. In the measuring room only two ladies happened to be at work. Quite a number of them are however employed in using the two micrometers and recording their measurements accurately in books. Baillaud's machine is used here also for measuring the magnitudes, all the three images of every star being used for this purpose. It will

be remembered that in the plan of the astrographic chart two plates are exposed upon each portion of the sky. The first plate s kept stationary as usual, but the second receives three adjustments which give every star three images at the vertices of a minute triangle the object being to eliminate accidental defects ot the plates. These images have had different exposures, and therefore have also different magnitudes, these three scales of measurement being then later on reduced to one by a simple computation. The positions of the stars are obtained by measuring each plate in two directions 180° apart, and that by the same lady in order that her personal index error may be eliminated. Both of these measurements are then independently repeated by another lady. In this way each plate is measured four times. As each star occurs on two plates, it follows that each star position rests upon eight determinations.

It was very instructive to see that in these methods of measuring intended for the general catalogue, not even the French observatories agreed among themselves. In Paris accuracy may be carried to its highest refinement on account of a large personell, in Bordeaux on the other hand speed is the chief object. Another very important experience gained by the visit was that in these French observatories this routine work of the astrographic catalogue and chart is by no means the only, no not even the principal occupation, but that each director is also following up his own ideas and employing assistants in them.

From France Father Hagen went to Holland where Father Stein, his assistant, joined him and accompanied him upon the rest of his trip. They first visited the Leyden observatory. A talk with its director was considered to be of great importance because he was always a prominent figure in Paris astrographic congresses, although his own observatory is not taking a practical part in the work. The director however was absent but his brother, E. F. van de Sande Bakhuysen, was present and could give complete information on all things desired. As the observatory and its numerous personell had been seen before, the new photographic equatorial was the only instrument examined. This differs from the usual astrographic type only in its greater focal length, because its object is to measure stellar parallaxes. The dome, the mounting and the electric outfit are all of the latest design. The new Repsold measuring engine, with which the plates are measured, is as excellent as the rest of the equipment. A personal conference with the director later on showed that he laid special emphasis upon the catalogue, and considered

that the chief result of the international astrographic enterprise, since the practical utility of the charts could hardly equal the enormous cost of their printing. In the catalogue itself the chief point was not the absolute, but only the relative positions of the stars, for the determinations of their proper motions.

In Groningen Professor Kapteyn called attention to the measuring engine designed by Sir David Gill, which probably succeeds best in uniting the greatest accuracy with speed in measurement. He also spoke of his plan of "Selected Areas" in the study of the sidereal problem, as he called it, that is, the structure of the sidereal universe. He looks at this problem not only according to Herschell's idea as an estimation or enumeration of the stars, but also examines their photometric and spectroscopic peculiarities, and their motions and distances. He had proposed this plan for the first time at the St. Louis World's Fair in September 1904, during the week of the Scientific Congresses, to a select gathering of astronomers, at which Father Hagen was present, and the writer also under his protecting aegis.

The way led next to England. At Greenwich the director Christie was absent at the time, but one of his assistants, Mr. Melotte, attended to the visitors and gave them a whole afternoon out of his off-duty time. There are two measuring machines each used by two men. They are of English make, by Grubb, and have instead of a micrometer screw a glass scale in the eyepiece, which enables the operator to keep his eyes adjusted to the same focal length while dictating his readings of the scale to an assistant. Parts of two plates are always measured at the same time in the same machine, because these overlap in the sky, so that each star is measured simultaneously on both plates. The micrometer scale is then reversed and the measures repeated. In this way it has been possible to nearly finish the Greenwich zone by this time. This large observatory has declined to print the charts according to the Paris plan. While the zone is indeed photographed according to the general plan and paper positives printed from the plates, the object of these prints is not their use by the observer at the telescope, but the ensuring of the plates themselves. The whole work is done at the observatory itself. The printed formulary for the registration of the measurements was examined with great care, as also the cases and presses in which the catalogues and the charts were stored away.

As the historic instruments of this old observatory had been inspected on a previous occasion, the present examination was confined to the new triple telescope. The great reflector on one

side of the axis is destined for sun spots and spectroscopic work. The other side supports a photographic telescope of about 65cm. aperture and a visual guiding telescope of 20 cm.

In Oxford the director Turner and his assistants, MM. Bellamy and Plummer Jr. devoted two days to the visitors. Their work consists only in the catalogue as they have on principle declined to construct the charts. The two measuring engines, each served by two young men, one dictating and the other writing, were likewise of English make, by Troughton and Simms, with a glass scale by Zeis of Jena. Instead of the magnitudes of the stars their diameters only are given, the formula for this conversion being given in the preface. Each plate is measured once in each of two opposite directions, by the same person, of course, as otherwise personal equation would vitiate the results. Professor Turner is satisfied with the lesser accuracy that a glass scale offers in comparison with a screw, because the completion of the work is as important to him as its accuracy. He looks upon a work that an observatory cannot complete in two decades, as daring and unpractical. In this view the history of all astronomical enterprises supports him. Thus the great Argelander, while contenting himself with a minimum accuracy in his Bonner Durchmusterung, was able to finish it in twelve years. Father Hagen himself has followed the same plan in his Atlas of Variable Stars, because he felt convinced that if this Atlas had not been finished in the last fifteen years, it would have remained forever unfinished. It is a noteworthy fact that Professor Turner is the only astronomer that has finished his share of the astrographic work in eighteen years. Next to him to finish will be the Greenwich Observatory. At other observatories the personell has already changed several times and will continue to change, on account of deaths and other causes, before the undertaking is brought to ân end. It is in this phase of the problem that the death germ of admiral Mouchez's great plan is much to be feared.

MM. Turner and Bellamy then conducted the visitors from their university observatory to the Radcliffe observatory, where in the absence of the director Mr. Wickham attended to them. With special pleasure he showed them the historic instruments of Pogson's time, because he had read in the *Monthly Notices* (No. LIX) Father Hagen's article on Pogson's intention to construct an atlas of variable stars. Besides these the new 60 cm. Grubb equatorial is specially interesting. It is photographic and destined for stellar parallaxes. The floor is raised and lowered

by electricity. The driving clock is perfect in its electric controlling mechanism, as also is the photographic camera at the eye end.

Leaving England the visitors proceeded to Brussels, to the large observatory at Uccle. The director Lecointe said he intended to take the zone of the astrographic charts which the Potsdam observatory has definitely declined. At this latter place the same as at the English observatories, the catalogue only of the zone was made without the charts.

As the German observatories had been visited before, Father Hagen contented himself on this trip with the two scientific congresses in Jena and Stuttgart. In Jena the astronomers were in session. They had come from Germany, Austria, Hungary, Denmark, Sweden, England, Mexico, Russia, Belgium, Holland and Switzerland. The city of Jena was chosen on account of its two optical institutions, the glass foundry of Dr. Schott and Co., and the glass grinding and instrument factory of Zeis. Both establishments were freely opened to the astronomers ,and guides were appointed to conduct them in groups and show them all the work. A large glass disk one meter in diameter was cast in their presence, the heat being so intense that the face had to be covered.

Besides attending to the public sessions, Fathers Hagen and Stein mixed much in colloquial intercourse with the astronomers in order to obtain their views and methods. The principal result gathered was that the German and English astronomers were generally opposed to the plan of the astrographic charts. As a fact only the four French observatories and the one Spanish one in San Fernando have as yet published a series of these charts. The northern astronomers thought that the expense of their construction was in an unfavorable proportion to their utility. On the other hand it was the general opinion that some zones might be finished with government help, but that the entire atlas would probably never be finished. If this is to be the fact, then the individual zones, even when completed, will lose much of their value, inasmuch as they will then be only fragments of a wrecked undertaking. Indeed, doubts were expressed as to whether even the French zones would ever be finished. The work in Paris is already under its third director, it has lasted nearly two decades and will require several more. Who can tell whether future directors will succeed in obtaining every year the enormous monetary appropriations necessary? More consequential than this is the question whether the next generation of astronomers will not find better methods for photographing the heavens and

then change Mouchez's plan. This has been the American idea from the start, and as a consequence not one of the many American observatories has joined in the French undertaking. Professor Pickering in Cambridge has as a fact simultaneously with Mouchez adopted a different plan, which is of much greater utility and at less expense. During the last two decades he has photographed the entire sky, from the north pole to the south pole, at least once in every two years. He does not measure and does not print the plates, but turns them into a reference library which contains the continuous history of the whole heavens. This photographic library has proved to be of inestimable value at the time of the discovery of the asteroid Eros, because Professor Pickering could at once furnish positions for it during previous years and thus compute its orbit directly. If this photographic library had existed at the time that Leverrier computed theoretically the position of the unknown planet Neptune, how easy it would have been to discover it and determine its orbit. There are many instances in which Professor Pickering could show for a newly discovered variable star a long series of previous maxima and minima sufficient to compute its period. He alone can prove whether or not such discoveries appertain to new stars, because he can at once examine the previous behavior of every suspicious object. And all this utility is at the cost of only the photographic plates and their development.

Such a limitation Kapteyn's plan also imposes upon itself, inasmuch as he wishes to repeatedly photograph only a relatively small number of "normal regions" of the sky. And on the plates themselves only a limited number of stars is to be measured. The new information that this method will give us concerning the structure of the starry heavens is beyond our ken at present. On the other hand the great astrographic atlas will show us on paper in one picture how the entire sky appears, from the north pole to the south pole, with all its stars from the first to the fifteenth or sixteenth magnitude, but after all it will not tell us more than how each part appeared at the time that a certain plate was exposed. It is not therefore a simultaneous picture of the heavens. Of proper motions, parallaxes, variations of light, new stars, this enormous atlas can tell us nothing.

The demand for such an atlas arose from the fine results achieved by the Durchmusterung of the heavens which Argelander had begun, and which Schönfeld and Thome afterwards continued. An extension of these charts from the 9½ to the thirteenth magnitude seemed to be necessary at the time that the asteroids were

being discovered. The Paris observatory had this problem in mind for a long time, but very wisely confined itself to the ecliptic zone. Chacornac began such on atlas, the Brothers Henry continued it, but found the work in the Milky Way alone too big for this zone. They then fell back upon photography and took new courage. If now wise limitation had confined its photographic work to the zone of the ecliptic, the atlas would be finished today and would satisfy every desire in the discovery and following-up of the small planets. It was probably owing to the enthusiasm which the success of the new photographic processes aroused, that not only the Paris observatory, but also the international congress which it had convened, committed themselves to an enterprise, the magnitude and difficulties of which they realized only afterwards.

If the original plan of charting the entire heavens is to be considered impossible to execute, three solutions to the problem may be suggested. The first would be to confine the work to an ecliptic zone ten or twelve degrees in width. Such an ecliptic atlas containing all stars down to the fourteenth or fifteenth magnitude might yet perhaps be collected into one large volume, and would probably constitute one of the finest of the astronomical achievements of the Paris Observatory.

A second solution of the difficulty would be to induce those observatories that have given up the charts, to finish their share of the photographic work and, according to Pickering's plan, to stow the plates away in library form. These plates would then be a true chart of the heavens, that could be consulted in the case of new discoveries. It would of course then be very desirable to unite all these partial libraries in one place, and for this the Paris observatory would seem to be most apt, since the idea originated there. This photographic collection would not indeed be a history of the heavens, as that of the Harvard observatory is, it would however give several magnitudes of stars more.

This second proposal leads directly to the third. If it should prove successful to induce the observatories that are taking part in the enterprise, to finish at least their photographic work and send their plates to the Paris observatory, it might then perhaps become possible to obtain the means to print these more than ten thousand charts. This of course would bring us face to face again with the old difficulty, but with this difference, that more unity would be put into the undertaking, a quality which it seems to lack since Mouchez's death.

So much for the astrographic chart. It is well known that the

plan of the international congress was a double one, and contained also a star catalogue besides the charts. The Durchmusterungen of Argelander, Schönfeld and Thome also consist of catalogues besides the charts, and indeed principally of catalogues. As a matter of fact these catalogues are used almost exclusively without the charts. Thome's charts are seldom taken in hand, and the photographic Durchmusterung of the Cape observatory consists essentially in its catalogue alone without charts. These catalogues were considered to be of such importance by astronomers that they were observed anew by common agreement, and furnished the so-called Astronomische Gesellschaft's catalogues. These last are not yet however completed, because the united action of many observatories seems to raise almost insurmountable obstacles. These Astronomische Gesellschaft's catalogues ought now to be extended from the ninth to the eleventh magnitude according to the Paris plan. This is done by means of the astrographic catalogues. These are approved of by all astronomers, and considered to be a very meritorious scientific work. No observatory that is at all able to do continuous work has let this part of the astrographic working plan fall through. Its completion seems indeed to be reserved for a future generation, but may probably be considered assured.

In order not to extend this article beyond reasonable dimensions, I must omit recounting Father Hagen's attendance at the congress of naturalists at Stuttgart, and his visits to the observatories of Padua, Venice and the Roman College, which had no direct bearing upon the astrographic chart.

Omaha, Nebraska.

THE SOLAR APEX. II.

THOMAS CURRAN RYAN.

For Popular Astronomy.

In my first article on this topic the suggestion was hazarded, that continued use of the method of study hitherto employed is not likely to cause any marked change in the average, or mean, of the results thus far obtained. This suggestion must be qualified by excepting from it the two results obtained by Oscar Stumpe from fourth and eighth magnitude stars, and the one obtained by Comstock from stars of magnitudes nine and a half and ten. Their apex results are marked four, eight, ten, upon the

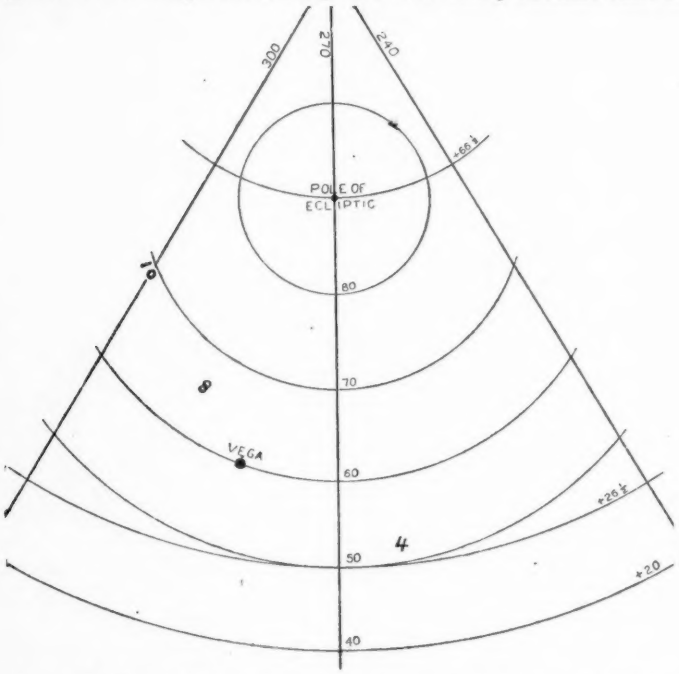
map accompanying this article. The numerals indicate the star magnitudes from which they were, respectively, obtained.

This putting of the question to a group of stars selected exclusively from one magnitude is a new departure in the search for the solar apex. Although a larger number of results from this method will be needed to give us a definite idea as to their meaning, yet the three already found are sufficiently suggestive to warrant a hope that by this method we shall get results that can be better interpreted than those obtained from groups of varying magnitudes. The latter may be termed the old method, and the results it has given us are, in a sense, like the stars consulted—unlike, and more or less scattered over a region of the sky forty two degrees long and sixteen wide. When we reflect that forty degrees is one ninth the total circumference of the heavens, it is apparent that we are far from a scientific solution. Philosophy may step in and note apparent tendencies, or suggest a mean for these varying answers to the riddle, but even philosophy is not satisfied with inferences drawn from data so questionable. It was high time to make a new departure.

The inquiry was judiciously begun by the elder Herschel. He gave it the right impulse. Whatever course the search may take, ultimately, it was well to lay its foundation in the study of star motions. By following in the path which he pointed out we have learned three all important facts:—1, that the Sun is moving through space; 2, that his velocity is, approximately, twelve and one half miles per second; 3, that his path is directed toward some point in or adjacent to that portion of the sky where solar apex results have been obtained by the study of star motions. As to whether that point is fixed in that portion of the heavens, or moves over some regular path therein, no theory has been advanced.

But though the old method has brought us a large sum of knowledge, I venture to doubt its ability to give us much more. It must be obvious that the effort to study the question from observations of star motions has great limitations. If the truth we are seeking can be revealed only by the motions of stars we are confronted by the fact that at least half of the stars are so distant that their motions cannot be observed. The pertinency of this observation is vouched for by the fact that different groups of stars have made unlike answers to our questioning. Hence we cannot fairly expect to know the whole truth until all these starry witnesses have spoken, or, at least, until we learn what the stars too far away to speak thus to us would say, if they could.

Shall we ever know what these silent witnesses could tell us? I venture to hope we shall, and that the present generation of astronomers will probe their secrets by following in this new path blazed out by Stumpe and Comstock. If the ten highest magnitude stars, questioned separately, shall fill out the line of apices foreshadowed in their three results, and if this line shall disclose a law of contour and of progression of the apex from magnitude to magnitude, may we not safely assume that the



Arcs of the larger circles indicate equatorial declination. Arcs of the smaller circles indicate parallels of ecliptic declination. The distances of the three apex results 4, 8 and 10, from the pole of the ecliptic are as follows:

4	36	degrees
8	26	degrees
10	21	degrees

fainter stars will take up this line and complete it? We should in that case, have a basis for further inference. In this work the local star drift, detected by Comstock, can be eliminated, and a study of southern stars can be made effectual for correcting displacements toward the equator.

But, suppose this new method will disclose such a line of apices. What will it mean? How shall we interpret the fact? The star motions from which apices are derived do not occur at the time the observations are made. They occurred at periods, in the past, more or less remote. Let us see if we cannot obtain some sort of an approximation to the time when the star motions actually took place from which were obtained the three results already mentioned.

If a star shines with faint light this may not be caused by great distance. Altair is a first magnitude star, yet is four times as distant as 61 Cygni, which shines with no more than fifth magnitude. Stars vary both in intrinsic brilliancy and in size, so that faintness of light may be exhibited due to these causes. Nevertheless, faintness always results from the increasing distance of any source of light. We have every reason to believe that variations in size and intrinsic brilliancy are pretty evenly distributed throughout the star system. A vast majority of the stars are very distant, and very faint. These considerations compel astronomers to believe that when a star shines with faint light the probabilities are many to one that its faintness is due to distance.

Professor Young in his "General Astronomy" says:

"As to the distance of the remotest stars in the stellar system, it is impossible to say anything very definite, but it seems quite certain that it must be at least as great as 10000 or 20000 light years."

The present state of our knowledge on this subject indicates that the time elapsed since the star motions took place from which results four eight and ten, on the map, were derived, is about as is shown in the following table:

Apex point on map	Observer	Magnitude of stars used	Antiquity of motions: years
4	Stumpe	4	160
8	Stumpe	8	450
10	Comstock	10	750

Now look at these figures, and at the map where the distances in degrees between these three apex results can be seen. They bear about the same relation to each other as do the magnitudes and the times elapsed since the motions took place. A considerable number of apex results obtained as these were—each from a group of stars of the same magnitude, would enable us to infer whether this coincidence is merely fortuitous or indic-

ative of some law of tendency. Shall we finally learn that the phenomenon of a shifting apex is real, and hence disclosed by consulting star motions that took place at several periods considerably removed from each other, in time? Or shall we learn that it is only an apparent phenomenon, caused by consulting star motions, which, viewed from our station in the universe, must show a ratio and contour affected by their distance?

Thus the scientific imagination will take its momentary flights, while the patient searchers for truth are laying the foundation of facts upon which alone science can rear its conclusions. Facts are stubborn things that bend to no man's theory, and, as enjoined by Thomas Huxley, we should "sit down before them like little children, prepared to follow humbly wheresoever they may lead." And there are some facts, well known, which *may* have an important bearing upon this search for the true solar apex, but which, if they are relevant, have not, so far as I know, been considered by those engaged in attempting to solve the problem.

I am, and for some time past have been, engaged upon a study of the direction of the Sun's path from the stand point of the orbit of the Earth. Not having yet arrived at a conviction I shall here say no more concerning this line of study than seems necessary to enable others to pursue it if they desire to do so.

Some fundamental facts, simple and well established, may be stated at the outset:—

Every phenomenon of celestial mechanics consists of a compound motion derived from the following forces and conditions:—

FORCES. I. Simple molar motion, including herein the speed, or time rate of motion. II. The attraction of gravitation, including herein the masses of the inter-attracting bodies.

CONDITIONS. I. Distance; II. Direction of the simple motion. III. The angle between the lines of simple motion and attraction.

Before going any farther, and in order to prevent misunderstanding of my purpose, let me make a brief quotation from Professor Young's "General Astronomy", and a brief comment thereon. Referring to the translation of the Sun and his attendant bodies through space, he says:—

"It is to be noted that this swift motion of the solar system, while of course it affects the real motion of the planets in space, converting them" (meaning their orbits) "into a sort of corkscrew spiral*****does not in the least affect the relative motion of Sun and planets, as some paradoxers have supposed it must."

So far as I know, Professor Young may be entirely right in this statement. The "relative motion" of a planet, of which he

speaks; that is, its motion as related to the Sun's motion, may be shown by the customary diagram of an ellipse, and the fact of an elliptical *ring-orbit*, thus indicated, may be *relatively* as true as it would be actually true if the Sun were fixed in space. And for many purposes, indeed for most purposes, it is more convenient to treat the orbit from this relative point of view. But there are some facts, and some elements of a planet's orbit that can be more vividly conceived and understood from a contemplation of the helicoidal, or corkscrew-spiral path in which a planet actually moves. At least, it has been much easier for me to understand the causes of the phenomena of varying distance, varying velocity, and eccentricity, and their relation to each other, from a study of the actual orbit. This, however, is a subject which I cannot enter upon here. The relevancy of the helecoidal path of a planet to a study of the direction of the Sun's path, if it has any relevancy to that study, lies in the fact that six of the planets revolve about the Sun in planes almost parallel to the plane of the ecliptic in which the Earth revolves.

I know no better way to introduce the reader to this line of thought than that in which it gradually came to me. It began with a conception of an ideally perfect orbit in which there is no such thing as varying distance, varying speed, or change of contour in the orbital curve,—in other words, no element of eccentricity. Such an orbit may be conceived from a study of figure 1, which is a pattern that can be used to form a model of an ideal, or perfect planetary orbit. The original from which it is reproduced was drawn upon a scale of eighty million miles to one inch. The reproduction here is, considerably smaller, but the proportions are preserved in it. It's width represents 584,000,000 miles, being the circumference of the Earth's relative orbit. Its height, 392,000,000 miles is a year's journey of the Sun, adopting what seems to be the most generally accepted velocity—that found by Campbell, twenty kilometers, or nearly twelve and one half miles per second. The diagonal line represents 703,000,000 miles, being the actual length of journey which the Earth must take in order to start from point 1 and reach point 5, upon a straight line through point 3.

Now if a piece of pasteboard is cut out on this scale, according to this pattern, ruled diagonally as in the figure, and the right and left edges brought together so that the paper is changed into a cylinder, the diagonal line, left on the outside, will be changed into a spiral and will show the actual orbit the Earth

would move over in one year's journey, provided the conditions were *all* as here represented. A hat pin, pencil or knitting needle, held upright in the center of the paper cylinder would represent the Sun's path. The diagonal line, when the paper is laid out flat, represents what must be the tangential impulse of the Earth's motion in order to permit this orbit. Nothing is shown as to the direction of this tangential impulse except its angular relation to the line of the Sun's path, when extended over and beyond that path. Obviously the velocity (and in this is included both speed and direction) must be such that the Sun's attraction can hold the planet constantly at a certain distance. Under

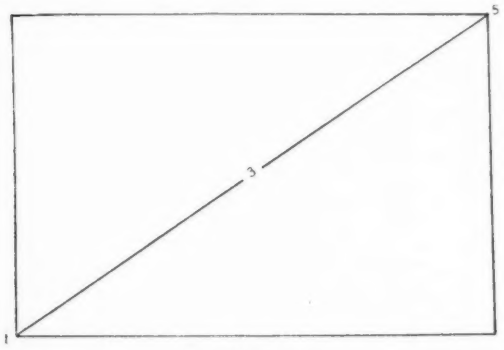


Figure 1

these conditions the spiral into which the diagonal straight line is changed, when the paper is formed into a cylinder, would represent a compound of the planet's velocity and the Sun's attraction.

It will be apparent that the office of the Sun's attraction, in the conditions assumed in figure 1, would not be to give the Earth either positive or negative acceleration of speed, but to change the *shape* of its orbit from the straight line of its original impulse of simple motion to the spiral line which that straight line assumes when *warped* by the Sun's attraction. In other words the attraction of the Sun would operate upon the path of the Earth as a helm does upon a ship's course. A perfect steamer with helm hanging free, sailing, as does a planet, without wind or water currents, would move in a straight line. But hold the helm steadily to port or starboard and there is a constant stress upon one side of the vessel retarding the motion of that side and causing the ship to move in a curve. The stress of the Sun's

attraction seems to have a similar effect in producing the compound motion represented by the orbit of a planet.

Now in this ideal condition, represented by the model formed from figure 1, there could not possibly exist any such element in the orbit as variation in distance between Earth and Sun, or acceleration or retardation of the Earth's velocity in any part of its path, or change of contour in the curvature of that path. All these would be the same, yesterday, today and forever, since there would exist no cause that could produce change in them.

The cylinder formed, as above directed, from figure 1, may be conceived as a cylinder of space around which the Earth would move if it had this perfect orbit. *It is a right cylinder, resting upon the plane of the ecliptic. Consequently this orbit would remain always in that plane.* Now the orbit of most of the planets are almost in this plane. Hence the cylinders of space around which their spiral orbits are twined must stand nearly upright upon that plane. That is to say, if the Sun's course inclines away from a perpendicular to that plane, the plane must cut the cylinder obliquely. In other words the cylinders of space around which the orbits run would, in that case, be oblique cylinders. Remember that this is necessary for the reason that the planets do stay in that plane, and it is only by changing the cylinder from right to oblique that they can be kept in that plane if the Sun's path be thus inclined. Does not every increase of obliquity of the cylinder as related to the plane involve an addition of eccentricity to the orbit of a planet *revolving in that plane*? In the right cylinder, an orbit without eccentricity is possible, though the planet remains always in the same plane. Bend the Sun's course down until it points toward Alpha Libra, or any other point upon the sky line of the ecliptic, and the only way that the Earth could remain in its present orbital plane would be by moving in a straight line. Between these two extremes lie all shades of obliquity, and consequently all shades of eccentricity, but no place for an orbit devoid of eccentricity, yet situated in the present plane of the ecliptic. Hence, in order to understand the fact of such nearly perfect orbits as are possessed by the planets which remain in that plane we must conceive of the solar apex as being very close to the pole of the ecliptic. Are these considerations invalid? I must submit that question to those better able to judge than myself. The fact that I have not been able to find any flaw in them is no proof that they cannot be overthrown.

Perhaps the question has never been considered whether this

perfect orbit *in the plane of the ecliptic* or anything so closely approaching it as most of the planetary orbits do, could be conceived as possible if the Sun's path was much inclined from a line perpendicular to that plane. When we conceive the Sun as moving along such perpendicular line, we have no difficulty in further conceiving any sort of existing orbit of planet or comet accompanying him upon such a journey. Upon this line all orbits from the ideally perfect one above supposed down to the most eccentric comet's path are certainly possible. And, obviously all these orbits are equally possible, no matter what direction the Sun's path may lie in. But if we conceive the Sun as moving upon a path even slightly inclined from a perpendicular to the plane of the ecliptic, can we find for it an orbit *in that plane* yet possessed of no eccentricity? My investigation thus far seems to show that though a perfect, or nearly perfect orbit, such as the Earth, or Venus has, would be as possible in that case as it would be if the Sun's path were perpendicular to the ecliptic plane, yet such an orbit would, *in that case*, carry the planet out of the plane in which it now moves, so that it would be in that plane only while crossing it at two periods of the year, instead of being constantly in it, as now. If this is true then the situation and direction of extension of the plane in which those planets having nearly perfect orbits revolve around the Sun would seem to have a significant bearing upon the question of the direction in which the Sun is moving—or, in other words, the solar apex. That plane certainly is no shifting thing. Its position is as fixed and enduring as the constellations of the heavens within whose circle it lies.

The translation of the Sun and his family through space, toward the solar apex must be slowly lifting the plane of the ecliptic out of the circle of the Zodiac. In the long run its place in the sky will be marked by other constellations. But this slow movement,—so slow that it has not yet been noticed, would have no effect upon the question here suggested.

THE GOLDSCHMIDT LAW OF COMPLICATION APPLIED TO THE SOLAR SYSTEM.*

WILLIAM HERBERT HOBBS.†

FOR POPULAR ASTRONOMY.

There are two distinct methods by which natural laws may be discovered. In the study of phenomena which are susceptible of investigation by physical methods of experimentation, correspondences may be observed. When the phenomena may not be subjected to experiment and observation, it may still be possible to discover fundamental laws through the fitting of number systems to the known facts; thus securing the expression of a formula by considering the common features of a sufficient number of special cases. The probability that a law thus discovered is correct, will depend upon the number of cases available for the test and upon the number of coincidences found, a probability which may sometimes be expressed by a mathematical figure.

Such a law is the fundamental periodic law of chemistry, in its crude form discovered by the late Professor Mendelejeff and elaborated by Lothar Meyer and others. Almost universally accepted as a fundamental law of the science, it was discovered through an arrangement of the numbers corresponding to the atomic weights of the element into groups of sevens with intermediate groups at regular intervals. It was found that such a system of groups could be fitted with much perfection to the known facts regarding the properties of the chemical elements. The deep significance of the law could not be doubted, albeit its physical basis is even now but vaguely outlined. Based upon its suggestions scientific prophecy has been in a quite remarkable manner fulfilled.

* This article has been based upon a study of Professor Goldschmidt's papers and upon many extended conversations with him upon the subject treated. The material has been drawn from several papers, and has been given a new arrangement as well as a somewhat different treatment. Professor Goldschmidt has accepted the Kant-Laplace theory so far as the original state of nebular matter is concerned. Inasmuch, however, as the Goldschmidt theory is in its essential parts quite independent of this or any other theory concerning the state of matter from which the universe was evolved; and since Chamberlin has shown the improbability of evolution of the planets out of highly heated gases: it has seemed best to omit some features of the Goldschmidt theory. Professor Goldschmidt is, of course, in no way responsible for statements here made; and the article must be regarded as a review into which the writer's personal views have entered.

† University of Michigan.

The problem of the origin of the solar system is clearly one to which experimentation can be applied only with great difficulty, and its laws are likely to be discovered, if at all, by the less direct method. The new hypothesis .of Goldschmidt* was suggested by a law of numbers which he found to have expression in the growth of crystals through the control of the occurrence upon them of definite faces. This so-called *law of complication* (identical with the law under consideration) grew out of the greatest systematic study of crystal forms ever undertaken†, and brings into one the four fundamental laws of crystallography. The law might almost with equal aptness be termed the law of simplicity, since it declares that the probability of the occurrence of any given face upon a crystal is controlled by the simplicity of its projection ratio within a harmonic system ot numbers‡.

Goldschmidt's methods of investigating crystals have thus greatly simplified crystallography, and the invention of the "Two circle goniometer" has widened the domain of the science much as have the great modern equatorials the knowledge of the heavens. This little laboratory at the University of Heidelberg has attracted students from all over the world, but Americans and Canadians especially have come under the tutelage of the genial German Crystallographer.

The harmonic system of numbers which characterizes the law of complication of crystals, applies, as Goldschmidt quickly saw, to the harmony in combination of phenomena depending upon vibration—musical tones and color¶. By applying the harmony theory to music it has been found possible as never before to analyze upon a scientific basis the accords of great compositions. Palestrina's *Stabat Mater*, which Helmholtz declared was inexplicable upon his musical theory though undoubtedly a harmonious composition, has been shown to have a very simple structure when tested by the law of complication.

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* Victor Goldschmidt, Ueber Harmonie im Weltraum. Ostwald's Annalen der Naturphilosophie (Leipzig), Vol. 5, 1906, pp. 51-101.

† Victor Goldschmidt, Index der Krystallformen. Vol. 1, 1886, pp. 601; Vol. 2, 1890, pp. 546; Vol. 3, 1891, pp. 420.

‡ Victor Goldschmidt, Ueber Entwicklung der Krystallformen. Zeitsch. f. Kryst., Vol. 28, 1897, pp. 1-35, 414-451. Ueber Verknüpfung der Krystallpartikel, ibid., Vol. 29, 1898, pp. 37-53. Ueber nicht parallele Verknüpfung der Krystallpartikel, ibid., pp. 361-385.

¶ Victor Goldschmidt, Ueber Harmonie und Complication. Berlin, 1901 pp. 113. Harmonische Analyse von Musikstücken, Ostwald's Ann. d. Naturphilosophie, vol. 3, 1904, pp. 449-508. An English outline of the musical theory has been printed by Richards (The Goldschmidt theory of harmony, Jour. Franklin Inst., Vol. 156, 1903, pp. 230-236; 301-315.).

The above indicates that the law of complication may be one of the universal laws in Nature, and the reason will be obvious. The broader problem which Professor Goldschmidt has set himself, is the course of harmonious development of the complex out of the simple. There are but four mathematical processes by which the nature of a body may be changed—by addition, by subtraction, by multiplication, and by division. In reality there are but two processes, addition and division, since subtraction is the addition of quantities of opposite character and multiplication the addition of equal quantities. If the source of increased complexity be carried back to the forces which produce it, subtraction in Nature is the addition of forces of opposite direction (A, Fig. 1); multiplication the addition of equal forces acting in

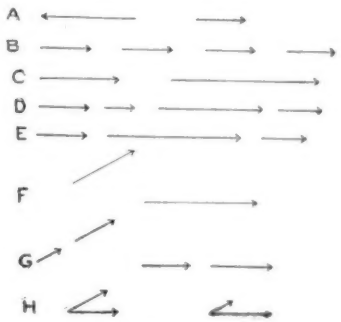

Figure 1.

the same direction (B. Fig. 1). It can further be shown that complexity cannot increase without the process of division, since it is only through division that the number of units can be increased. Moreover, without addition no combination can occur, and increased complication must be assumed to arise from new combinations. But to divide forces of a single kind (single direction) and to again combine them can hardly increase complication, since the resulting forces would still be of one kind as before (C, D, and E, Fig. 1). Division and subsequent addition of unlike kinds (forces of different directions) are therefore essential to the development of complication (F, G, and H, Fig. 1).

We are seeking the nature of the evolution from the simplest possible to the next most complex by a method which produces harmony, is regulated by a simple law. Our processes of division must thus be assumed to be the simplest possible, and the additions likewise the simplest possible. Let two forces A and B

(Fig. 2) act in different directions which meet at an angle. The simplest possible division of these forces is obtained by halving them, and their combination by addition is the resultant C; which with the original forces makes the *first complication.* We have now a derived force C, which may be halved and combined in the same manner with the original forces again halved, to produce a *second complication* (A D C E B). A *third complication*

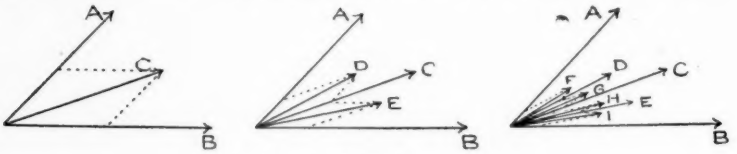

Figure 2.

results from the combination by a similar process of D with A and C, and of E with B and C (AFDGCHEIB). The same process may be continued to later complications, but it is clear that the subsequently derived complications will, like all late terms in series which involve increasing powers of a fractional quantity, be relatively unimportant and therefore negligible.*

Of the derived forces, that of the first complication clearly stands next in importance to the original forces, and its name both in crystallography and in music—the *dominant*—has been adopted for the general law.

We are in need now of some mathematical method of expression for each of the derived forces, and one is found in the *gnomonic projection* already in use in crystallography. If in Figure 3,

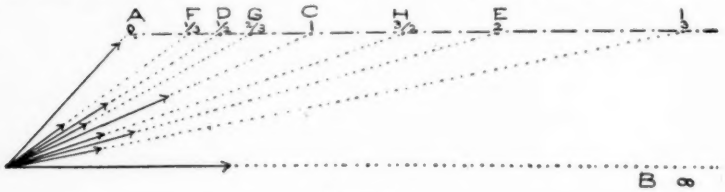

Figure 3.

A and B be the original forces, and a line be drawn parallel to B, the several derived forces will be projected upon it by their intersections with it. The intersection of the original force A will be zero,

* For simplicity of application this is most important, since otherwise the question might arise whether a combination after division by three might not be simpler for the first complication than one after successive halvings in late complications.

and that of the other original force is ∞. The combination of these in the first complication—the dominant— we will make unity; which makes the derived forces of the second complication 1/2 and 2, those of the the third complication 1/3, 2/3, 3/2, and 3 respectively. It thus appears that the terms to the right of the dominant are the reciprocals of those to the left, as is also clear from the figure. Such a series as is represented by any complication might be described as a *Brocot Series** (the portion 0 to 1) to which an inverted Brocot Series has been added. The writer would suggest that the new series be known as the *Goldschmidt Series*.

The relative importance of the terms of a Goldschmidt Series is that of their order of development, thus;

$$\underset{\equiv}{0} \quad \underset{\equiv}{1/3} \quad \underset{=}{1/2} \quad \underset{=}{2/3} \quad \underset{\equiv}{1} \quad \underset{=}{3/2} \quad \underset{=}{2} \quad \underset{=}{3} \quad \underset{\equiv}{\infty}\dagger$$

The terms of a Goldschmidt series express only ratios, and if in some other than the *normal form* (limits 0 and ∞) may be transformed from a series with limiting members z_1 and z_2 provided only the relative position of any term in the series (z) is not changed. The formula for tranformation is, therefore;

$$P = \frac{z - z_1}{z_2 - z}$$

where p is any term of the normal series and z the corresponding term of the original series. If all the terms of a normal series of the third complication are represented, the series is known as a *complete normal series*. It will be evident from the bi-lateral symmetry of the Goldschmidt series that its presence in any series of numbers will first appear when the dominant or central member has been found. This is accomplished by multiplying or dividing the series as a whole by such numbers as will make different members successively equal to unity; which process cannot destroy the integrity of the original ratios.

* See E. Cohen, Théorie des nombres, Paris, 1900, p. 333.

† Mathematically the later complications may be derived by adding numerators and denominators of adjacent terms of the next earlier complications when these are expressed as fractions, thus:

First Complication $\frac{0}{1}\left(\frac{0+1}{1+0}=\frac{1}{1}\right)\frac{1}{0}$

Second Complication $\frac{0}{1}\left(\frac{0+1}{1+0}=\frac{1}{2}\right)\frac{1}{1}\left(\frac{1+1}{1+0}=\frac{2}{1}\right)\frac{1}{0}$

Third Complication $\frac{0}{1}\left(\frac{0+1}{1+2}=\frac{1}{3}\right)\frac{1}{2}\left(\frac{1+1}{2+1}=\frac{2}{3}\right)\frac{1}{1}\left(\frac{1+2}{1+1}=\frac{3}{2}\right)\frac{2}{1}\left(\frac{2+1}{1+0}=\frac{3}{1}\right)\frac{1}{0}$

The universe and especially that part best known to us, the solar system, must be conceived to have originated in a simple form and structure which became differentiated through the operation of some physical law or laws—*developed harmonious complications*. In the till lately generally accepted Kant-Laplace hypothesis, a homogeneous mass of highly heated vapor is gradually cooled and condensed with separation successively of annular equatorial rings which ultimately change to rotating spheroids following the paths once outlined by the rings. Recently Chamberlin* has discovered such serious defects in the hypothesis that it is no longer regarded with the same reverence as heretofore. Chamberlin's planetesimal theory of the universe conceives of the planets within the solar system as originating through the aggregation of so-called discrete particles, and he is able to explain upon this basis the internal planetary heat as well as the atmospheres of the larger bodies. The motions of these meteoric bodies are not irregular and uncontrolled as would be necessary to reproduce the conditions within a coarse gas as predicted by Lockyer and Darwin, but are restrained within orbital paths like the planets which are evolved from them.

The fact that Saturn's ring is today generally regarded as made up of a great number of small solid bodies which follow a nearly common orbit like that of a satellite, supports this hypothesis. The recent important contribution of seismology that the Earth's core transmits earth-waves like a rigid body and with a velocity one and a half times as great as would the hardest steel, gives further support to Chamberlin's view and confirms the statement of Lord Kelvin that the Earth's rigidity must be greater than that of a ball of glass of equal size in order successfully to resist the tidal stresses.

As a collection of related planetary bodies within a system of which the Sun is the center, the solar system presents a number of well determined facts which must stand in some relation to the origin of the system and must be taken into account by any cosmical hypothesis. These data relate to:

1. The relative volumes of a) the Sun and planets, and b) tributary bodies—satellites.
2. The relative specific gravities of the Sun and the planets.
3. The directions of axial and of orbital rotation.
4. The periods of axial and of orbital rotation.

* T. C. Chamberlin, a group of hypotheses bearing on climatic changes. Jour. Geol., Vol. 5, 1897, pp. 653-683. Also T. C. Chamberlin and R. D. Salisbury, Geology, Vol. 2, 1906, pp. 1-132.

5. The eccentricity of the ellipsoids of form—the flattening of the poles.

6. The relative distances of the planets from the Sun.

7. The relative distances of the satellites from their parent planets.

So far as the data above referred to for the Sun and planets are numerical, they are expressed in the following table, the order being that of distance from the Sun.

	Diameter in kilometers	Flattening of poles	Mean distance from the Sun (Earth distance=1)	Period of orbital rotation (in years)	Period of axial rotation (in hours)	Specific gravity (average of values of Humboldt and of Meyer)
Sun	1386690	?			612	1.42
Mercury	4816	?	0 3871	.24		5.65
Venus	11969	?	0.7233	.61		5.41
Earth	12756	1:299	1.0000	1.00	24	5.56
Mars	6745	1:210	1.5237	1.88	24.6	3.99
Jupiter	143757	1:14	5.2028	11.86	9.9	1.31
Saturn	119075	1:11	9.5389	29.46	10.3	.72
Uranus	59171	1:10	19.1834	84.02		.80
Neptune	54979	?	30.0567	165.1		1.17

A mere glance at the above table reveals the fact that we have to do, not with a single series of related planets, but with two series which may be distinguished as those of the small and of the large planets respectively. How great is the disparity of size of the first series—Mercury to Jupiter—is made clear by Figure 4.

Figure 4. Diagram to show the relative sizes and the positions of the members of the solar system.

The position of the Sun is indicated by the star at the left. The upper series shows the actual positions, the lower the positions of the planets as calculated by the law of complication.

If the large planets be regarded as born of the Sun, the small planets belong to a different and probably a later generation. No less distinctly are the two series differentiated in respect to their specific gravities. The planets of the inner and smaller series have densities ranging from 4 to 5½, while the large planets all have densities below 1.31. Except the so-called lighter alkali metals, we know of few such light bodies save only the solidified gases of the atmosphere. The meteorites which reach the Earth have the average specific gravity of 5.4, and the conclusion is therefore drawn that they are derived not from distant space but from the region of the nearer planets.

The polar flattening of the inner planets appears to be of a

The universe and especially that part best known to us, the solar system, must be conceived to have originated in a simple form and structure which became differentiated through the operation of some physical law or laws—*developed harmonious complications*. In the till lately generally accepted Kant-Laplace hypothesis, a homogeneous mass of highly heated vapor is gradually cooled and condensed with separation successively of annular equatorial rings which ultimately change to rotating spheroids following the paths once outlined by the rings. Recently Chamberlin* has discovered such serious defects in the hypothesis that it is no longer regarded with the same reverence as heretofore. Chamberlin's planetesimal theory of the universe conceives of the planets within the solar system as originating through the aggregation of so-called discrete particles, and he is able to explain upon this basis the internal planetary heat as well as the atmospheres of the larger bodies. The motions of these meteoric bodies are not irregular and uncontrolled as would be necessary to reproduce the conditions within a coarse gas as predicted by Lockyer and Darwin, but are restrained within orbital paths like the planets which are evolved from them.

The fact that Saturn's ring is today generally regarded as made up of a great number of small solid bodies which follow a nearly common orbit like that of a satellite, supports this hypothesis. The recent important contribution of seismology that the Earth's core transmits earth-waves like a rigid body and with a velocity one and a half times as great as would the hardest steel, gives further support to Chamberlin's view and confirms the statement of Lord Kelvin that the Earth's rigidity must be greater than that of a ball of glass of equal size in order successfully to resist the tidal stresses.

As a collection of related planetary bodies within a system of which the Sun is the center, the solar system presents a number of well determined facts which must stand in some relation to the origin of the system and must be taken into account by any cosmical hypothesis. These data relate to:

1. The relative volumes of a) the Sun and planets, and b) tributary bodies—satellites.
2. The relative specific gravities of the Sun and the planets.
3. The directions of axial and of orbital rotation.
4. The periods of axial and of orbital rotation.

* T. C. Chamberlin, a group of hypotheses bearing on climatic changes. Jour. Geol., Vol. 5, 1897, pp. 653-683. Also T. C. Chamberlin and R. D. Salisbury, Geology, Vol. 2, 1906, pp. 1-132.

5. The eccentricity of the ellipsoids of form—the flattening of the poles.

6. The relative distances of the planets from the Sun.

7. The relative distances of the satellites from their parent planets.

So far as the data above referred to for the Sun and planets are numerical, they are expressed in the following table, the order being that of distance from the Sun.

	Diameter in kilometers	Flattening of poles	Mean distance from the Sun (Earth distance=1)	Period of orbital rotation (in years)	Period of axial rotation (in hours)	Specific gravity (average of values of Humboldt and of Meyer)
Sun	1386690	?			612	1.42
Mercury	4816	?	0 3871	.24		5.65
Venus	11969	?	0.7233	.61		5.41
Earth	12756	1:299	1.0000	1.00	24	5.56
Mars	6745	1:210	1.5237	1.88	24.6	3.99
Jupiter	143757	1:14	5.2028	11.86	9.9	1.31
Saturn	119075	1:11	9.5389	29.46	10.3	.72
Uranus	59171	1:10	19.1834	84.02		.80
Neptune	54979	?	30.0567	165.1		1.17

A mere glance at the above table reveals the fact that we have to do, not with a single series of related planets, but with two series which may be distinguished as those of the small and of the large planets respectively. How great is the disparity of size of the first series—Mercury to Jupiter—is made clear by Figure 4.

Figure 4. Diagram to show the relative sizes and the positions of the members of the solar system.

The position of the Sun is indicated by the star at the left. The upper series shows the actual positions, the lower the positions of the planets as calculated by the law of complication.

If the large planets be regarded as born of the Sun, the small planets belong to a different and probably a later generation. No less distinctly are the two series differentiated in respect to their specific gravities. The planets of the inner and smaller series have densities ranging from 4 to 5½, while the large planets all have densities below 1.31. Except the so-called lighter alkali metals, we know of few such light bodies save only the solidified gases of the atmosphere. The meteorites which reach the Earth have the average specific gravity of 5.4, and the conclusion is therefore drawn that they are derived not from distant space but from the region of the nearer planets.

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wholly different order from that of the larger and lighter bodies. Only 1:200-030 in the former. It is 1:10-14 in the latter. Thus it appears that the two series are in all the respects of size, density, eccentricity of form, and period of axial rotation, sharply differentiated from each other; and hence probably originated at different times. This is the first great conclusion from Goldschmidt's study of the solar system. The relative sizes and the distribution in space of the planets within each series are, however, made the subject of more detailed examination.

An additional distinguishing characteristic of the series of large planets is the abundance of satellites. Jupiter has seven, Saturn ten, and a system of rings, Uranus four, and Neptune one.

Within the group of the four larger planets, the two larger bodies have almost the same diameter, as have also the two smaller bodies. The same is true of the group of smaller planets, and the proportion of the average large to the average small planet is identical for the two series, as will be clear from the following table:

	Diameters		
Jupiter	143757 kil.		
		mean 131416	
Saturn	119075 "		
			$\frac{131416}{57075}=2.3$
Uranus	59171 "		
		mean 57075	
Neptune	54979 "		
Venus	11969 kil.		
		mean 12363	
Earth	12756 "		
			$\frac{12363}{5781}=2.1$
Mercury	4816 "		
		mean 5781	
Mars	6745 "		

Other interesting relationships estab-

Figure 4

lished for each of the two systems in common will be treated after each system has been tested by the law of complication.

The distances from the Sun of the larger planets are as follows:

DISTANCES FROM THE SUN.

	IN MILLIONS OF KILOMETERS			AFTER DIVIDING BY 1466			FRACTIONAL EQUIVALENTS
	min.	max.	mean	min.	max.	mean	
Jupiter	735.9	810.6	773.2	0.502	0.553	0.522	½
Saturn	1338.3	1497.3	1417.8	0.918	1.016	0.967	1
Uranus	2719.2	2983.5	2851.3	1.855	2.036	1.945	2
Neptune	4429.6	4505.5	4467.5	3.022	3.075	3.048	3

The Sun being the zero point from which our measurements are taken, it is evident that we have here to do with a Goldschmidt series of numbers, namely:

Sun		Jupiter		Saturn		Uranus	Neptune	Space
0	.	1/2	.	1	.	2	3	∞

The division of the distance measured in millions of kilometers by the factor 1466 has brought the dominant to the planet Saturn, and it is found that the series of numbers is transformed into a Goldschmidt series of the third complication with three members (1/3, 2/3, and 3/2 missing). From the relative sizes of these planets it was evident that only Jupiter or Saturn could have been considered the dominant.

The smaller planets, of which Venus and the Earth are much the largest (like Jupiter and Saturn in the larger series) are distributed as follows:

DISTANCES FROM THE SUN.

	IN MILLIONS OF KILOMETERS			AFTER DIVIDING BY 733			FRACTIONAL EQUIVALENTS
	min.	max.	mean	min.	max.	mean	
Sun							
Mercury	45.6	69.4	57.5	0.062	0.095	0.078	$\frac{1}{13} = 0.0077$
Venus	106.7	108.3	107.5	0.146	0.148	0.147	$\frac{1}{7} = 0.143$
Earth	146.2	151.1	148.7	0.199	0.206	0.203	$\frac{1}{5} = 0.200$
Mars	205.4	247.6	226.5	0.280	0.338	0.309	$\frac{1}{3} = 0.333$
Jupiter	735.9	810.6	773.2	1.004	1.106	1.055	$1 = 1.000$

In this instance our series is not found in the normal form, but between the limits 0 and 1, namely:

Sun	Mercury	Venus	Earth	Mars	Jupiter
0	1/13	1/7	1/5	1/3	1

To transform into a series of the normal form we must employ the general transformation formula

$$p = \frac{z - z_1}{z_2 - z}$$

which for our special case becomes

$$p = \frac{z}{1 - z}$$

and we obtain:

Sun	Mercury	Venus	Earth	Mars	Jupiter
0	1/12	1/6	1/4	1/2	∞

Venus and the Earth are of about equal size, and multiplication by six or by four will make Venus or the Earth respectively the dominant. The series obtained are, it is worthy of note, in each instance a Goldschmidt series, and the author of the theory here speaks of a probable *twin dominant.*

With Venus dominant (multiplication by 6) we have:

Sun	Mercury	Venus	Earth	Mars	Jupiter
0	1/2	1	3/2	3	∞

With Earth dominant (multiplication by 4) we have:

Sun	Mercury	Venus	Earth	Mars	Jupiter
0	1/3	2/3	1	2	∞

It remains to examine in a similar manner the satellites within the series of larger planets. For all save Neptune the number is sufficient for such an examination. The known data are given in the following table.

SATELLITES.*

	Diameter in kilometers	Mean Distance from parent planet in thousands of kilometers	Distance after factoring	Nearest rational value
JUPITER			(Factor 970)	
Io	3935	419	0.44	1/2 = 0.50
Europa	3260	666	0.70	2/3 = 0.67
Ganymede	5696	1064	1.11	1 = 1.00
Callisto	5359	1871	1.95	2 = 2.00
URANUS.			(Factor 417)	
Ariel		205	0.49	1/2 = 0.50
Umbriel		285	0.68	2/3 = 0.67
Titania		443	1.06	1 = 1.00
Oberon		583	1.40	3/2 = 1.50
SATURN.			(Factor 1228)	
Mimas		184	0.15	1/7 = 0.14
Enceladus		236	0.19	2/11 = 0.18
Thetis		293	0.24	1/4 = 0.25
Dione		378	0.31	1/3 = 0.33
Rhea		523	0.43	2/5 = 0.40
Titan	2400?	1215	0.99	1 = 1.00
Hyperion		1475	1.20	6/5 = 1.20
Japetus		3539	2.88	3 = 3.00

For the satellites of Jupiter and Saturn the Goldschmidt series appear at once and are respectively:

Jupiter	Io	Europa	Ganymede	Callisto	Space
0	1/2	2/3	1	2	∞
Uranus	Ariel	Umbriel	Titania	Oberon	Space
0	1/2	2/3	1	3/2	∞

* The newly described and fainter satellites for which the data are less definite have not been included in the list.

The diameters of the satellites of Uranus are not known, but Titania is the brightest, and it will be interesting to see if it is also the largest in case the relative sizes should be determined. Saturn has been placed last in the table, instead of in its natural order, because of the great number of its satellites and the evidence that we have here to do with a combination of two systems dating from different periods of aggregation, as was found to be true of the planets themselves. We have not, however, in this case the full evidence there found, since specific gravity, polar flattening etc., have not been determined; but the evidence from relative sizes and from the orbital diameters is very striking. The most important satellite of the series is Titan, the first to be discovered, which occupies a position similar to that of Jupiter within the system of planets, and there is evidence of the presence of a series between the orbit of this body and Saturn. Transforming the series in exactly the same manner as was done for the series Sun-Jupiter and shifting the dominant to Thetis (multiplying by 3) we obtain:

Saturn		Mimas	Enceladus	Thetis	Dione	Rhea		Titan
0	.	1/2	2/3	1	3/2	2	.	∞

The remaining satellites are less clearly in correspondence with a Goldschmidt series, since the little Hyperion, which is the faintest of the entire series and was discovered from 100 to 200 years later than the others falls but poorly into correspondence. Were its rational number 2 instead of 6/5 the reality of the series would be apparent thus:

Saturn				Titan		Hyperion	Japetus	Space
0	.	.	.	1	.	(2) 6/5	3	∞

Despite this badly fitted term the analogy between the planets (the first and second generations) and the satellites (the third and fourth generations) is very striking. Goldschmidt suggests that from analogy we may look to find the outer satellites of Saturn having a different specific gravity and rotational period than the inner series.

If we make Saturn the dominant or middle point of the series of larger planets and the Earth that of the inner series; we find that the ratio of the diameters of dominants for the two series is identical with the ratio of their orbital radii, thus:

$$\text{Diameters} \quad \frac{\text{Saturn}}{\text{Earth}} \quad \frac{119075}{12756} = 9.3$$

$$\text{Diameters} \quad \frac{\text{Saturn}}{\text{Earth}} \quad \frac{1417.8}{158.7} = 9.5$$

If now we consider each series in its entirety by concentrating the total volume at the position of the dominant we obtain a similar but slightly larger value for the ratio:

$$\frac{V}{v} = \frac{D^3}{d^3} = \frac{(D)^3}{(d)^3} = \frac{143757^3 + 119075^3 + 59171^3 + 54979^3}{4816^3 + 11969^3 + 12756^3 + 6745^3} = 1195.7 = 10.6^3$$

$$\therefore \frac{D}{d} = 10.6$$

Hence the diameter of the combined spheres is as the distance from the Sun.

It is thus found that the law of complication applies throughout the solar system, as is shown by the following recapitulation:

Normal Goldschmidt series	0	1/3	1/2	2/3	1	3/2	2	3	∞
Series of larger planets	0		1/2	.	1	.	2	3	"
Series of smaller planets	0	1/3	.	2/3	1	.	2	.	"
Satellites of Jupiter	0	.	1/2	2/3	1	.	2	.	"
" " Uranus	0	.	1/2	2/3	1	3/2	.	.	"
" Saturn (Inner series)	0	.	1/2	2/3	1	3/2	2	.	"
" " (Outer series)	0	.	.	.	1	'.'	(6/5)	3	"

It will probably suggest itself to some that certain of the gaps might have been occupied. The group of asteroids falls in the space between Mars and Jupiter ($2 - \omega$) in the inner series of the planets, while the small bodies together composing the rings of Saturn fall in a reciprocal position ($0 - 1/2$) in the inner series of the satellites of Saturn. Goldschmidt believes, however, that the asteroids and Saturn's rings belong in a different category which he has designated accessories and for which the explanation must be sought in the works cited.

His examination of the Cosmos in the light of the law of complication Goldschmidt concludes by a citation from Kant's *Cosmogonie* written in 1763:

"Although my statement is crude and unelaborated I hope to say something which may open for others the way to important considerations. Something of it has in my opinion a degree of probability which in the case of a matter of less importance would leave little room for doubt, and against which only the assumption of a greater artfulness than we generally ascribe to the fundamental laws of nature can be set up.

Ann Arbor, Mich.,
May 1, 1907.

ON SOME SERIOUS ERRORS DISCOVERED IN THE KINETIC THEORY OF GASES AND IN THE APPLICATION TO IT OF NEWTON'S SECOND LAW OF MOTION, PROVING THE CORRECTNESS OF D'AURIA'S FUNDAMENTAL EQUATIONS OF THE ABOVE THEORY.

LUIGI D'AURIA.

FOR POPULAR ASTRONOMY.

The present investigation was undertaken by the writer to justify certain new results pointed out by him in his paper "On the kinetic theory of gases" read at the New York Meeting of the American Association for the Advancement of Science, December 30, 1906, and published in *Popular Astronomy*, April 1907, under the title "A new development of the kinetic theory of gases." This new development was based upon the following new theorem contributed by the author.

The kinetic energy of a smooth, hard and perfectly elastic sphere which is forced to oscillate between two perfectly elastic parallel plates perpendicular to the direction of its motion while one of these plates is moving towards the other with uniform speed which is very small compared with that of the sphere, varies inversely with the length of the path described by the sphere while oscillating between the plates.

Observing that in a gaseous medium composed of smooth, hard and perfectly elastic spheres, compression shortens the mean free path of these spheres proportionately to the change of volume, and that this shortening which takes place quite slowly comparatively to the speed of the spheres, is attended by an increase of kinetic energy of translation of the spheres, the only kind of energy they can acquire, the writer was led to the conclusion that this increase of energy should take place in accordance with his theorem, and thus, in a gaseous medium as described above we should have

$$\bar{e}s = \text{constant}, \qquad (1)$$

in which $\bar{e}$ and s are respectively the mean kinetic energy and the mean free path of each sphere, which quantities, in accordance with the kinetic theory, are the same for all the spheres constituting the medium.

Since $\bar{s}$ in the same medium is proportional to the volume V which the medium occupies, we can write

$$\bar{e}V = \text{constant}, \qquad (2)$$

If now we consider each series in its entirety by concentrating the total volume at the position of the dominant we obtain a similar but slightly larger value for the ratio:

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Satellites of Jupiter	0	.	1/2	2/3	1	.	2	.	"
" " Uranus	0	.	1/2	2/3	1	3/2	.	.	"
" Saturn (Inner series)	0	.	1/2	2/3	1	3/2	2	.	"
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$$\bar{e}s = \text{constant}, \qquad (1)$$

in which $\bar{e}$ and s are respectively the mean kinetic energy and the mean free path of each sphere, which quantities, in accordance with the kinetic theory, are the same for all the spheres constituting the medium.

Since $\bar{s}$ in the same medium is proportional to the volume V which the medium occupies, we can write

$$\bar{e}V = \text{constant}, \qquad (2)$$

and also

$$EV = \text{constant}, \qquad (3)$$

in which E is the kinetic energy of agitation of the medium, and from this relation the writer deduced

$$dE = E\frac{dV}{V},$$

in which dE is the increase of kinetic energy attending a diminution of volume dV of the medium in a vessel impervious to energy. Denoting by p the pressure which the medium exerts upon unit area of the inner surface of this vessel, he pointed out that we should have

$$pdV = dE = E\frac{dV}{V},$$

or

$$pV = E \qquad (4)$$

from which

$$p = \frac{E}{V} = \frac{\frac{1}{2}M\overline{u^2}}{V} = \frac{1}{2}\rho\,\overline{u^2}, \qquad (5)$$

in which M is the mass of the gaseous medium, $\overline{u^2}$ is the mean square of the speeds of its spheres, and ρ is the density of the medium. Comparing (4) with (3) the conclusion

$$pV^2 = \text{constant} \qquad (6)$$

was reached, which the writer compared with the well-known thermodynamic equation

$$pV^\gamma = \text{constant} \qquad (7)$$

and concluded that γ, which is the ratio of the two specific heats of the gaseous medium considered, must be equal to *two*.

In the development of the kinetic theory of gases by Clausius, Maxwell, Boltzman and other eminent investigators, Newton's Second Law of motion is made use of in order to find the expression for the gaseous pressure p due to the impacts of the spheres constituting a kinetic gas. This very fundamental law of Mechanics, first known to Galileo, is, in the words of Newton: "Mutationem motus *proportionalem esse* vi motrici impressæ, et fieri secundum lineam rectam qua vis illa imprimitur"; which, in terms of modern science, means that change of momentum *is proportional to force*, and is in the direction in which the force acts. As a given change of momentum mv can take place in different periods of time, it can be readily seen that the force must vary inversely with the time t in which the change of mo-

mentum occurs; and therefore, if the average value of the force in time t is (ϕ), we should have

$$(\phi) \propto \frac{mv}{t}, \tag{8}$$

or,

$$t\,(\phi) = kmv, \tag{9}$$

in which k is a co-efficient to be determined.

In the problem of bodies moving against a constant resistance, as in the case of bodies projected vertically from the surface of the Earth with moderate velocities, it can be shown that $k = 1$, and for all such cases we have

$$t(\phi) = mv. \tag{10}$$

But we cannot apply this equation to the case of the impact of a perfectly elastic, smooth sphere to determine the pressure it produces against the surface impinged upon without first examining whether in this case also the co-efficient k is equal to *one*. This has been disregarded by every physicist who has been dealing with the problem of the pressure due the impact of elastic bodies, and Newton himself, in his investigation of the problem of the pressure due to the impact of the particles of a liquid jet against a plane surface, applied equation (10). With this assumption it is found that the pressure on a section of unit area in a gas composed of smooth, hard and perfectly elastic spheres is measured by the amount of momentum transferred by the spheres in unit time across the section, computed in a direction perpendicular to the section. Thus, if $\overline{v}$ is the mean square speed in this direction for the spheres crossing the section, m the mass of a single sphere, and Q the number of spheres crossing the section in unit time, we should have, in accordance with (10),

$$p = Qm\overline{v}. \tag{11}$$

In the usual treatment of the kinetic theory the value of Q in this equation is made out to be equal to $N\overline{v}$, in which N is the number of spheres contained in unit volume of the gas. Thus it is found

$$p = Nm\overline{v^2} = \rho\,\overline{v^2} \tag{12}$$

in which ρ is the density of the gas; or, expressing $\overline{v}$ in terms of the mean square speed $\bar{u}$ of the spheres irrespective of direction,

$$p = \tfrac{1}{3}\rho\overline{u^2}, \tag{13}$$

which is the familiar expression for the gaseous pressure, and

which differs considerably from our equation (5) in its numerical factor.

From equation (13) it follows

$$pV = \tfrac{2}{3}E \qquad (14)$$

leading to the well-known theoretical ratio of the two specific heats $\gamma = \frac{5}{3}$. These results which since the kinetic theory was mathematically developed have been considered by physicists as being identified with Newton's Second Law of motion, are based in reality upon an assumption which, as it will be shown later on is not correct. But this is not the only error which I have found in the kinetic theory. A very careful examination of the problem involved in the computation of the value of Q has disclosed to me that this value should be $N\bar{u}$, and not $N\bar{v}$. This will become clear when we observe that in the section of the gas considered each sphere may be supposed to belong to a string of spheres moving inclined to the plane of the section at any angle with speed equal to the mean square speed $\bar{u}$, and that the distance between two contiguous spheres on each string may be supposed to be equal to the average distance between two contiguous spheres in the whole mass of gas which can be expressed by $1/N^{\frac{1}{3}}$. Thus observing that the number of strings occupying the section is $N^{\frac{2}{3}}$, the number of spheres which in unit time must be transferred across the section must be

$$Q = N^{\frac{2}{3}}\left(\bar{u} \div \frac{1}{\sqrt[3]{N}}\right) = N\bar{u}.$$

Substituting in (11) the value of Q we have just found we get

$$p = Nm\bar{u}\,\bar{v} = \rho\bar{u}\bar{v}; \qquad (15)$$

and assuming as before

$$\overline{v^2} = \tfrac{1}{3}\overline{u^2},$$

we would have

$$p = \frac{1}{\sqrt{3}}\rho\,\overline{u^2} \qquad (16)$$

which is larger than our value (5), and very much larger than the value (13) generally admitted by physicists.

We have assumed that $\bar{v} = \bar{u}/\sqrt{3}$. Now, if we consider the matter more carefully we shall find that this relation is not quite correct. The directions of the motions of the spheres which move across any very small portion of a section of unit area taken in a mass of gas, if observed with reference to the plane

of the section for a unit of time, they would be found to comprise every angle between *zero* and 90 degrees, and any one of these angles would be found to be repeated in the same interval of time as many times as any other in the series. Hence, it seems to me that the average component of the mean free path $\bar{u}$ at right angles to the plane of any section of the gas should be equal to the mean of $\bar{u}\sin\psi$ taken between the limits $\psi=0$ and $\psi=\frac{1}{2}\pi$. Thus the average component in question would be

$$\frac{\int_0^{\frac{1}{2}\pi}\bar{u}\sin\psi d\psi}{\frac{1}{2}\pi}=\frac{2\,\bar{u}}{\pi}=\bar{v}. \qquad (17)$$

With this value, which is a little larger than $\bar{u}/\sqrt{3}$, equation (15) gives

$$p=\frac{2}{\pi}\rho\overline{v^2} \qquad (18)$$

which is even larger than the value (16), and would lead to the ratio of specific heats

$$\gamma=1+\frac{4}{\pi} \qquad (19)$$

which is larger than my value $\gamma=2$.

We have, so far, carried our investigation upon the assumption that $t(\phi)$ is equal to mv. The problem before us now is to examine whether this assumption is valid for the case of the impact of a smooth, hard, perfectly elastic sphere against a fixed surface. It will be observed that during the impact of the sphere the momentum mv of the sphere at right angles to the plane changes twice. The first change takes place while the sphere is having its kinetic energy transformed into potential energy, and the second while this energy is changed back again into kinetic energy. Each transformation requires the same time t, and therefore the average force (ϕ) developed during the impact must act upon the surface for the time $2t$, in which time a total change of momentum $2mv$ occurs.

In order to be able to visualize all the details of the above problem, let the impinging sphere be a smooth spherical shell devoid of mass and absolutely inalterable, at the center of which, condensed into a comparatively very small volume is balanced a spherical mass m in such a manner that its displacement from the center of the shell in any direction shall be elastically resisted by a force represented by the equation

$$\phi=A\,x^n, \qquad (20)$$

in which x is the radial displacement, n is an exponent to be determined and A is a constant co-efficient.

It will be noticed that the shell having no inertia is bound to adjust itself concentrically to the mass m when the system is moving in free space. In other words, when so moving, the system is utterly unable to acquire or retain internal energy, a condition which is essential to secure perfect elasticity. When the shell impinges upon the fixed surface its internal mass m begins its radial excursion inside of the shell, moving in a direction normal to the surface with initial speed v which at some distance X from the center of the shell becomes *zero*. Thus we see that the distance X must be described by the mass m with a mean speed (v) which is always smaller than the speed v possessed by the sphere normally to the fixed plane. Of course, it will be understood that in order to prevent the mass m from striking the shell, we must have in every case of impact

$$\tfrac{1}{2} m v^2 < \int_{x=0}^{x=R} A\, x^n\, dx$$

in which R is the radial displacement which the mass m will have to describe in order to touch the shell.

From what has been specified above we can now write the following equations:

$$(\phi)\, X = \tfrac{1}{2} mv^2, \text{ and } X = t(v)\ ,$$

in which (ϕ) is the mean value of ϕ given by the equation (20). Eliminating X, we get

$$t\,(\phi) = \tfrac{1}{2} \frac{mv^2}{(v)} = \tfrac{1}{2} \frac{v}{(v)} \cdot mv,$$

which equation compared with (9) gives us the general expression for the co-efficient k, that is,

$$k = \tfrac{1}{2} \frac{v}{(v)}. \qquad (21)$$

The following analysis will enable us to express this co-efficient in terms of the displacement X. Let ω denote the speed of the mass inside of the shell at the end of the displacement $x < X$. Then the amount of kinetic energy which the mass m will have lost at the end of this displacement can be expressed by

$$\tfrac{1}{2} mv^2 - \tfrac{1}{2} m\omega^2 = \int_{x=0}^{x=X} \phi\, dx - \tfrac{1}{2} m\omega^2$$

and this loss of energy must be equal to the work done by the force ϕ over the path x, which work is

$$\int_0^x \phi\, dx\ .$$

Hence,

$$\tfrac{1}{2} m\omega^2 = \int_{x=0}^{x=X} \phi\, dx - \int_0^x \phi\, dx = \frac{A}{n+1}\left(X^{n+1} - x^{n+1}\right),$$

and

$$\omega = \frac{dx}{dt} = \left[\frac{2A}{(n+1)m}\left(X^{n+1} - x^{n+1}\right)\right]^{\frac{1}{2}};$$

from which we get

$$t = \left[\frac{(n+1)m}{2AX^{n+1}}\right]^{\frac{1}{2}} \times \int_{x=0}^{x=X} \frac{dx}{\sqrt{1-\left(\frac{x}{X}\right)^{n+1}}}\ . \qquad (22)$$

Observing that

$$A\int_{x=0}^{x=X} x^n dx = \frac{AX^{n+1}}{n+1} = \frac{1}{2} mv^2\ ,$$

we can easily see that

$$\left[\frac{(n+1)m}{2AX^{n+1}}\right]^{\frac{1}{2}} = \frac{1}{v}\ ,$$

which substituted in (22), gives

$$t = \frac{1}{v}\int_{x=0}^{x=X} \frac{dx}{\sqrt{1-\left(\frac{x}{X}\right)^{n+1}}}$$

and

$$v = \frac{1}{t}\int_{x=0}^{x=X} \frac{dx}{\sqrt{1-\left(\frac{x}{X}\right)^{n+1}}}\ .$$

Multiplying and dividing this equation by X, observing that $X/t = (v)$, we find

$$\frac{v}{(v)} = \frac{1}{X}\int_{x=0}^{x=X} \frac{dx}{\sqrt{1-\left(\frac{x}{X}\right)^{n+1}}}\ . \qquad (23)$$

Concerning the value of n, we must remember that this is the exponent of x in equation (20) by which the elastic resistance encountered by the mass m is represented. This resistance, in accordance with Hooke's law, must be proportional to the displacement, and therefore we must put $n = 1$. Thus we find

$$\frac{v}{(v)} = \frac{\pi}{2}, \tag{24}$$

which substituted in (21) gives

$$k = \frac{\pi}{4}, \tag{25}$$

and the latter substituted in (9) gives us the equation

$$t(\phi) = \frac{\pi}{4} mv, \tag{26}$$

which, for the first time in the history of Mechanical Science, expresses correctly the Second Law of motion as applied to the impact of perfectly elastic bodies.

Now it is easy to see that, in accordance with this new equation, the last value of p given by the equation (18) must be multiplied by k in order to obtain the correct expression for the gaseous pressure. We have then

$$p = \frac{\pi}{4} \times \frac{2}{\pi} \rho \overline{u^2} = \frac{1}{2} \rho \overline{u^2} \tag{27}$$

which is identical with the expression (5) found in my new development of the kinetic theory of gases, independently of Newton's Second Law of motion.

It may be observed that equation (10) which is based upon the assumption $k = 1$, is true when $v/(v) = 2$. This result follows from our equation (23) when $n = 1$. This reduces our equation (20) simply to $\phi = A =$ constant, which shows that equation (10) is applicable only when the body considered is moving against a constant resistance. For any other case, the equation

$$t(\phi) = \frac{1}{2} \frac{v}{(v)} mv \tag{28}$$

should be used, the ratio $v/(v)$ being determined according to circumstances.

If the new equation (26) expressing the relation between pressure and change of momentum in the impact of perfectly elastic bodies had been known at the time Clausius and Maxwell investigated the kinetic theory, the method pursued by them in re-

gard to the amount of momentum transferred by the molecules of a gas across unit area in unit time, would have led them to the result

$$p = \frac{\pi}{12} \rho \overline{u^2} \qquad (29)$$

and thus they would have arrived at the theoretical ratio of the specific heats

$$\gamma = 1 + \frac{\pi}{6} . \qquad (30)$$

This result, it will be observed, is considerably below the lower limit of the ratio of the specific heats of mercury vapor found experimentally by Kundt and Warburg which is 1.631, the higher limit being 1.695, and since in no case the theoretical value should be reached by any real gas, the result (30) would have proved the kinetic theory to be untenable, and perhaps would have induced physicists to examine carefully into the method above mentioned by which the amount of momentum transferred across unit area by the molecules of a gas in unit time is usually determined.

It would be very difficult to find in the annals of science a case parallel to the one we have been discussing. Here we have a theory which, in the words of a great physicist, was shadowed forth by Lucretius, definitely stated by Daniel Bernoulli, largely developed by Herapath, made a reality by Joule, worked out to its present advanced state by Clausius and Maxwell, and declared to be the greatest achievement yet made in molecular science. How could any one have imagined that the results of this theory, in its present advanced state, could be vitiated by such errors as have been pointed out above, and to involve besides the erroneous application of the Second Law of motion? Yet, this seems to have been the case, unless I have been laboring under an exceedingly strange delusion.

Perhaps no one will be able to appreciate the difficulties under which the above investigation has been carried out. The new results found in my development of the kinetic theory, which have been quoted at the beginning of this paper, have been viewed with the most discouraging suspicion by physicists on account of their supposed violation of the fundamental law mentioned above. The ratio of the specific heats $\frac{5}{3}$ has always been identified with this law, and a writer of note on the subject has said: "There is no modification of the kinetic theory as hitherto treated which could give a higher value for the ratio in

question than §." This is the opinion of every physicist to-day.

Referring to the factor one-half in my expression for the gaseous pressure, an able physicist says: "I have always been under the impression that the factor one third (in the usual expression) resulted necessarily from the fundamental principles involved. If the molecules of the gas are moving in all directions, and if they rebound at the walls of the confining vessel without loss of energy, the constant comes out necessarily one-third. Any other value would indicate a violation of Newton's Laws."

The above criticism is just what could be expected from any conservative, sound physicist, all the circumstances in the case being considered. It is to be hoped, however, that in view of the facts produced in this paper, my results concerning the kinetic theory of gases may now receive a fair consideration.

Philadelphia Penn.

THE MATHEMATICAL TABLETS OF NIPPUR.*

DAVID EUGENE SMITH.

Until the middle of the last century the mathematics of the Babylonians was practically a mystery to modern European scholars. It was asserted, it is true, in a general way by many writers on the history of the science that our sexagesimal fractions originated in the valley of Mesopotamia. Formaleoni of Venice had suggested as early as 1788 that these were due both to the ancient division of the year into three hundred and sixty days and to the properties of the regular hexagon. Even the Greeks had assigned these "fractiones physicæ," as they were called in the middle ages, to the Babylonians, and before their use by such writers as Hypsicles and Ptolemy the story had long been current that Pythagoras had studied mathematics with the priests along the Euphrates, where the sexagesimal system was known. There were also numerous assertions in the later middle ages that even our Hindu-Arabic numerals were due to the Chaldeans, the Kaldâ of the monument inscriptions, who overran and subdued Babylonia, in the first millenium, B. C. It was this tradition that led Tonstall to say, in 1522: "Qui a Chaldeis primum in finitimos, deinde in omnes pene gentes fluxit," and Recorde (*c* 1542) to remark: "In that thinge all men do agree, that the Chaldays, whiche fyrste inuented thys arte, did set these figures as thei set all their letters. for they wryte backwarde as you tearme it, and so doo they reade."

All this was mere fiction, or but little more than tradition, and our knowledge of Babylonian mathematics may be said to have had but little scientific foundation until Rawlinson deciphered two small and imperfect cylinders found

* *Mathematical, Meteorological and Chronological Tablets from the Temple Library of Nippur.* By H. V. Hilprecht. Vol. XX, part I, of Series A, Cunieform texts, published by the Babylonian Expedition of the University of Pennsylvania, 1906. Pp. xviii + 70 + (30,+ xv) plates. Price, $5.

in 1854 by W. K. Loftus at Senkereh, the ancient Larsam (Larsa), on the Euphrates, and now preserved in the British Museum. Discovering the meaning of the key word IB—DI, square, Rawlinson was able to show that these cylinders contained tables of squares and cubes, written on the sexagesimal system. Professor Sayce put the date of the cylinders between 2300 and 1660 B. C., and thought that there had been a great library at Senkereh which would probably yield other mathematical material. Unfortunately this expectation has never been realized so far as Senkereh is concerned, although a few other similar cylinders have been found elsewhere. There has recently been uncovered at Nuffar (the ancient Nippur), however, a collection of unexpected richness, and particularly valuable for the light which the cylinders throw upon ancient mathematics.

The excavations at Nippur began in 1889, and for eighteen years Professor Hilprecht has been connected with the work. He has examined with more or less care, over 50,000 cuneiform tablets thus far excavated there, and has secured for Constantinople and Philadelphia the best treasures of what may have been a great temple library which the Elamites twice destroyed, viz., about 2150 B. C. and about 1900 B. C. Some of these tablets are brick textbooks prepared by teachers and possibly deposited with other works in the temple library. Some have the teacher's model and the pupil's copy, and still others seem to show the erasure of the latter's work. At any rate, such are the dates and circumstances and interpretations which Professor Hilprecht assigns to his great discovery. There are not wanting, however, those who attack this view, some in a spirit of apparent fairness, and others with what appears to be a spirit of somewhat captious criticism. It is claimed that the tablets have no literary value, and that there is no evidence of a temple library, since some of the cylinders come from one part of the ancient city and some from another. One writer even weakens his cause by offering the remark that the temple may have had a system of Carnegie branches. It is also said that several important tablets did not come from Nippur at all, and an Arab's statement is taken against Professor Hilprecht's argument to prove this fact. While it would seem to a laymen in matters Assyriological that there may have been two libraries at Nippur, as there were at Nineveh, one old and the other of later date, and that the statement of an Arab trader would not have much weight in a scientific controversy, it is certain that the bickering over seventeen tablets out of some fifty thousand should not obscure the fact that here is the greatest discovery of all time relating to Babylonian mathematics. It is probable that, with the support of such eminent scholars as Professor Zimmern of Leipzig, Professor Hommel of Munich, and Professor Winckler of Berlin, Professor Hilprecht will not feel that his labors have been vain. With the whole controversy the student of mathematical history need have no concern whatever. Here are the cylinders; they are genuine; they are ancient; they reveal the science of Babylonia of the second or third millenium B.C., and whether there was a temple library, or whether an Arab told the truth, is from the mathematical standpoint a consideration of no moment.

It is the result of a study of over forty mathematical cylinders that appears in the volume under review. These cylinders include multiplication and division tables, tables of squares and square roots a geometric progression, a few computations, and some work on mensuration.

The multiplication tables are all arranged on the column plan, the same that was used by the medieval Italians in their "per colonna" operations. They also resemble the Italian tables in that the multiplicands are not successive, the European tables giving only those products needed for the current measures.

Professor Hilprecht feels sure that this reason did not, however, influence the Babylonian mathematicians, although this hypothesis might easily be in harmony with his discovery that the multiplicands are all factors of 60^4. The tables are carried much farther than those found in medieval Europe, the multiplicands extending to 180,000.

More valuable than the multiplication tables are those of division and of these the cylinders numbered 22 and 25 are the most interesting. In the former the quotients of 60^4 by 1, 2, . . ., 18 are given, as follows:

IGI 1 GAL–BI 8,640,000 A–AN
IGI 2 GAL–BI 6,480,000 A–AN
IGI 3 GAL–BI 4,320,000 etc.

The meaning will be understood by knowing that IGI–GAL (literally, "having an eye," hence the decider, determinator, denominator) means, practically, the divisor or simply "divided by"; that BI means "its," and refers to the dividend, which in this case is the mysterious 60^4; and that A–AN means "each." Hence the second line may be translated, "60^4 divided by 2 = 6,480,000 each." In other words we have here the equivalent of a table of unit fractions of 60^4, in which the first is not $\frac{1}{1}$, but $\frac{2}{3}$. Now why, asks Professor Hilprecht was $\frac{2}{3}$ thus indicated as a multiplier? In reply, it seems not unreasonable to consider the correspondence between these Babylonian remains and the Egyptian mathematics of the (nearly) contemporary Ahmes: This at once throws a good deal of light upon both of the cylinders here discussed. This No. 22 simply means the first fraction of 60^4 is 8,640,000; the second 6,480,000, and so on. Now this first fraction in the Egyptian hieroglyphic and hieratic systems seems, from Ahmes, to have been $\frac{2}{3}$, the only non-unit fraction known to him, and apparently the first one known to the Babylonians. It was the only one, save $\frac{1}{2}$, to have a special symbol in Egypt, and it probably had a special name, at least in, Babylonia. The very fraction name, GI–GAL, "having an eye," suggests the eye-shaped fraction symbol of the Egyptians, and raises the question whether the original might not have been an eye instead of *ro*, a mouth. Nevertheless it must be admitted that the tablet in question does not contain the well-understood cuneiform fraction symbols for $\frac{1}{2}$, $\frac{2}{3}$, etc., and that there is, therefore, no direct connection between it and the Ahmes manuscript. As to the fraction symbol, too, the relation between the form of eye and mouth is probably merely fanciful, although the Semitic (as distinguished from the Sumerian) Babylonian and the Hebrew both use "mouth" for fraction, as the Egyptians did. Thus the Babylonian for $\frac{2}{3}$ is *shînipu*, probably from *sinâ* (two) and *pû* (mouth), the "two-fraction," a form also found in Hebrew.

The most interesting of all the cylinders, however, is No. 25 which is transliterated as follows:

Line 1 :	125	720	Line 9 :	2,000	18
2 :	IGI–GAL–BI	103,680	10 :	IGI–GAL–BI	6,480
3 :	250	360	11 :	4,000	9
4 :	IGI–GAL–BI	51,840	12 :	IGI–GAL–BI	3,240
5 :	500	180	13 :	8,000	18
6 :	IGI–GAL–BI	25,920	14 :	IGI–GAL–BI	1,620
7 :	1,000	90	15 :	16,000	9
8 :	IGI–GAL–BI	12,960	16 :	IGI–GAL–BI	810

Of this Professor Hilprecht has, with much ingenuity, unraveled part of the meaning, viz: that $60^4 \div 103{,}680 = 125$, which accounts for two figures in lines

1, 2, and similarly for other pairs. He also notices that $125 = 2.60 + 5$, and that $3{,}600 \div 5 = 720$, and similarly for other odd numbered lines. Now is there any further explanation? In reply it may be noticed that the unit fraction already mentioned, a form met not only among the Egyptians but among the early Greeks and other peoples about the Mediterranean, may have played a part hitherto unrecognized among the Babylonians. This is also seen in the table on page 27, where certain unit fractions of 195,955,500,000,000 are given, and in numerous other places. Therefore instead of lines 1, 2, meaning 60^4: $103{,}680 = 125$, the interpretation may be as follows:

$$\frac{1}{60^2} \cdot \frac{1}{103{,}680} \cdot 60^2 = \frac{1}{60^2} \cdot 125 = \frac{1}{30} + \frac{1}{720}.$$

That is lines 1, 2 give the important numbers in connection with all these unit fractions, viz.: 125. 720, and 103,680, assuming of course, that the computer knew the nature of the problem with respect to the powers of 60. In the same way lines 3, 4 may mean

$$\frac{1}{60^2} \frac{1}{51{,}840} \cdot 60^4 = \frac{1}{60^2} \cdot 250 = \frac{1}{15} + \frac{1}{360}$$

and so on. In every case the denominator of the last unit fraction is given. *E.g.*, lines 7, 8 give the unit fractions $\frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{90}$: lines 9, 10 give $\frac{1}{2} + \frac{1}{8}$ and so on. All of the unit fractions save the last are readily seen, at least after the initial $\frac{1}{30}$ is found, so that only the last denominator seemed necessary. It is evident, however, that the cylinder does not actually contain these unit fractions, and the explanation here suggested may be far from the true one; but it is consistent, and if not exact it seems to show at least a connection between the number concepts of Babylonia and the Mediterranean countries. Moreover the cylinder has three other features of interest: (1) Passing from lines 7, 8 to 9, 10 the fractions (if we consider them as such) would become, following an apparent law of doubling, $\frac{8}{15} + \frac{1}{45}$; but, to maintain the unit fraction idea, they are written $\frac{1}{2} + \frac{1}{18}$ the denominator 18 appearing in line 9, all of which recalls the Egyptian treatment; (2) The table gives both an increasing and decreasing geometric progression; (3) It is evidently based upon $60^4 = 12{,}960{,}000$ which, as Professor Hilprecht notes, underlies all the mathematical texts described in this work, and which is nothing less than the mystic Platonic number, the "lord of better and worse births," the number of days in the "magnus Platonicus annus" of 36,000 years (of 360 days each), and the number at the basis of all the multiplication and division tables of Nippur, Sippar, and the library of Ashurbânapal.

The historic interest in this Platonic number does not relate to any extent to the supposed mysticism involved in it, but to the fact that Plato not improbably received it from the Pythagoreans, and they from their master the tradition of whose sojourn by the Euphrates is thus in some slight degree confirmed. At any rate, the whole matter seems to show a relation between the East and the West, both in the underlying idea of the unit fraction and in the number mysticism of the ancient philosophers.

A further debt which historians of mathematics owe to Professor Hilprecht arises from his contribution to the subject of ancient geometry. The results of his investigation of several cylinders on mensuration, together, no doubt, with the Scheil-Eisenlohr investigation* of the Ine-Sin tablet show "that at this early

* Ein altbabylonischer Felderplan nach Mittheilungen von F. V. Scheil, herausg. u. berab. von A. Eisenlohr, Leipsig, 1896.

period the Babylonians must have been familiar with the following theorems: 1. The area of a rectangle is equal to the product of its base and altitude. 2. The area of a square is equal to the square of its side. 3. The area of a right triangle is equal to one-half the product of its base and altitude. 4. The area of a trapezoid is equal to one-half the sum of its bases multiplied by its altitude." He also shows that they must have known either how to find the volume of a parallelepiped (and hence a cube) or else of a circular cylinder (and hence the area of a circle).

Still another inference of great importance, and supported by evidence that seems sufficient, is that the Babylonians knew, in some form or other, the law of the expansion of $(a + b)^2$, although whether this was derived through a study of geometric forms or by induction with numbers, it is impossible as yet to tell.

A final point of much interest may be metioned, viz., the fact that the subtractive principle of the Romans is also found in the Babylonian remains. The Romans spoke of nineteen as "undeviginti," writing it as IXX or XIX, and Professor Hilprecht gives no less than twelve cuneiform methods of indicating the same idea (20–1). It seems to be certain that the Babylonians applied this principle to 2 as well as 1, and even to higher numbers.

The work is illustrated by numerous drawings of mathematical tablets, and by a series of carefully executed photographic plates.

Altogether there has not appeared since the publication of the Eisenlohr translation of Ahmes such a valuable contribution in the way of source material for the study of ancient mathematics. It is earnestly to be hoped that Professor Hilprecht will continue in this important line, and that he will be able to assist still further in clearing up a number of vexed questions relating to the early mathematics of Mesopotamia. In particular it would be helpful if he could throw some light upon ancient calculation,* upon the Babylonian abacus (if one existed), and upon the relation (if any) between the number names of Phoenicia, Egypt, and Babylon. It is also to be hoped that he may succeed in giving us some information about the mathematics of the Shumeri(?), those non-Semitic inhabitants of the Euphrates valley whose language, the Sumerian, should, in the natural course of events, have influenced the mathematical terminology of much of the ancient world. It is to these people that there seems due some of the first work in mathematics and astronomy, and it is probable that the sexagesimal system itself had birth among them.

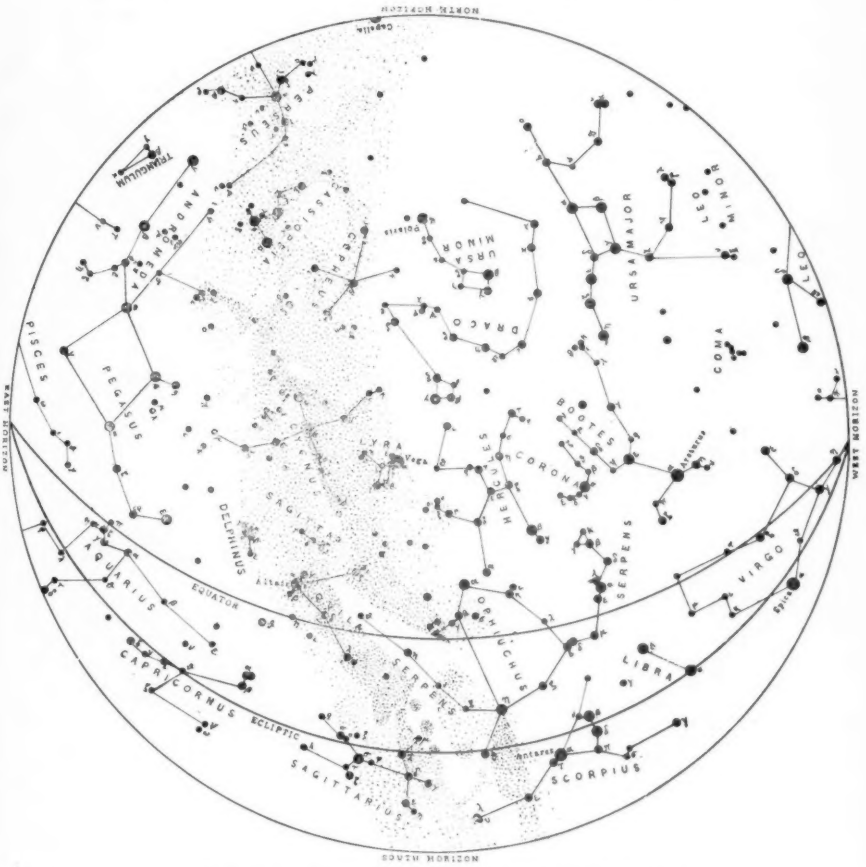
BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, May, 1907.

* His cylinder 25*a* may, when fully deciphered, contribute something.

PLANET NOTES FOR JULY AND AUGUST, 1907.

H. C. WILSON.

Mercury will be evening star during the first part of July but will not be conspicuous to the eye. He will come to inferior conjunction July 24 at 9:00



THE CONSTELLATIONS AT 9:00 P. M. AUGUST 1, 1907.

P.M. C.S.T., and will be visible as morning star during the middle of August. Greatest western elongation, 18° 51′ from the Sun, will occur on August 12. The planet will be at its brightest about a week later. Mercury will be in conjunction with Jupiter July 31 at 10:00 P.M. and with Venus on the next morning at 7. Mercury will be between four and five degrees south of both of these planets at the times of conjunction. Again on August 10 at 9:00 P. M., C. S. T.,

Mercury will be in conjunction with Jupiter, this time being 2° 05′ south of the great planet.

Venus is morning planet but is approaching conjunction with the Sun so that she will not be a conspicuous object during these months. On August 1 at 11:00 A.M. Venus and Jupiter will be in conjunction, Venus being then 0° 18′ north of Jupiter.

Mars will be at opposition on July 6 at 9:00 A.M. The planet's nearest approach to the Earth will be a few days later, on July 13, when its distance will be a little less than 38,000,000 miles. The apparent diameter of the disk of Mars will then be 24″.8, so that it will appear as quite a large object in the telescope and so the markings of its surface may be readily seen. The declination of Mars, however, will be so far south (−28°) that for northern observers the altitude of the planet at best will be too low for satisfactory observation except on rare occasions.

Jupiter will be in conjunction with the Sun July 15, so that its position is unfavorable for observations during these months.

Saturn may be seen after midnight in the constellation Aquarius. The plane of the rings is now so near the Earth that they are seen nearly edgewise, giving the appearance of a golden pin piercing through a golden ball. The satellites also move back and forth past the planet apparently in almost straight courses.

Uranus will be at opposition July 3 and so is in best position for study during the year. Uranus will be in conjunction with Mars, and 5° 18′ north of that planet, on July 19 at 1 P.M. Again on August 24 at 3 P.M., Uranus will be 4° 37′ north of Mars.

Neptune will be in conjunction with the Sun July 5 and so is not in position for study during the summer.

Occultations visible at Washington.

Date 1907	Star's Name	Magnitude.	IMMERSION. Washington M.T. h m	Angle f'm N. °	EMERSION. Washington M.T. h m	Angle f'm N °	Duration. h m
July 20	δ Libræ	4.4	5 26	169	6 08	241	0 42
21	B.A.C. 5746	6.2	11 07	102	12 20	274	1 13
23	Piazzi xix, 61	5.5	14 04	91	15 09	250	1 05
27	ψ^3 Aquarii	5.2	10 55	352	11 09	328	0 14
28	Piazzi 0^h,1	6.0	9 34	112	10 17	208	0 43
31	μ Ceti	4.3	13 43	24	14 35	283	0 53
Aug. 2	63 Tauri	5.7	14 19	127	14 54	191	0 35
18	58 Ophiuchi	4.8	10 53	77	11 59	285	1 06
19	28 Sagittarii	5.6	10 56	29	11 37	322	0 41
22	B.D.−17°6389	6.5	7 27	92	8 32	243	1 05

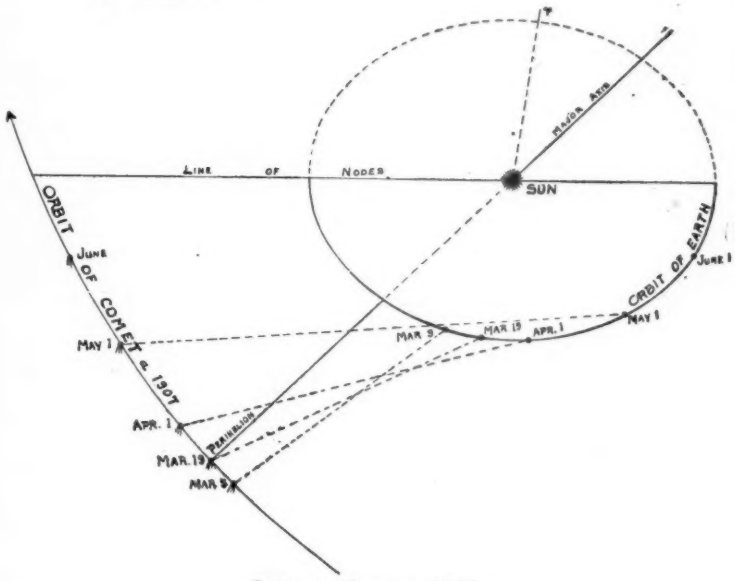
COMET NOTES.

Comet *a* 1907.—This comet is now very faint and is probably beyond the reach of all but the largest telescopes. The position and motion of the comet with reference to the Earth are shown in the accompanying diagram, which was prepared from the elements computed by Misses Einarson, Glancy and Joy of the Berkeley Astronomical Department. It will be seen that the comet was discovered near its perihelion and near the time of its closest approach to the Earth and that since that time the two bodies have been separating rapidly.

ELEMENTS OF COMET *a* 1907.

$T = 1907$ Mar. 19.22895 Gr. M. T.
$\omega = 317°\ 10'\ 07''.4$
$\Omega = 97\ 11\ 07\ .4$ 1907.0
$i = 141\ 39\ 15\ .6$
$q = 2.05160$

Dates of observations, Mar. 9, 13, Apr. 3, 9.



ORBIT OF COMET *a* 1907.

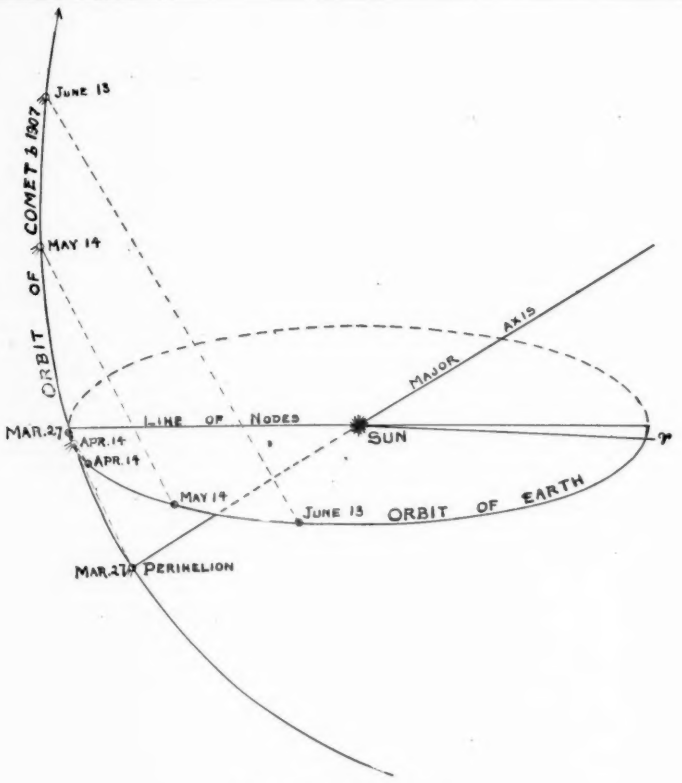
The position of the comet for May 29.5 is R. A. $6^h\ 16^m\ 19^s.1$, Decl. $+ 18°$ 35′ 13″ and its brightness one-fifth of that at the time of discovery.

Comet *b* 1907. The diagram of the orbit of this comet was prepared from the elements computed by Miss Lamson and Mr. Frederick at Washington, which satisfy the comet's movement surprisingly well for preliminary elements computed from observations only one day apart. It will appear from the diagram that the comet was moving roughly perpendicular to the orbit of the Earth and that it was near its point of closest approach to the Earth when discovered. If it had passed perihelion a few days earlier the approach to the Earth would have been very close indeed. Since that time the two bodies have been separating rapidly and the comet will probably not be visible at all in June. On May 8, the comet was found near the predicted place, but was very faint as seen with our 16-inch telescope. Clouds prevented a measure of its position.

ELEMENTS OF COMET *b* 1907

$T = 1907$ Mar. 27.56 Gr. M. T.
$\omega = 328°\ 47'$
$\Omega = 189\ 07$
$i = 110\ 12$
$q = 0.924$

Dates of observations. Apr. 15, 16, 17.



ORBIT OF COMET *b* 1907

VARIABLE STARS.

Minima of Variable Stars of the Algol Type.

[Given to the nearest hour in Greenwich Mean Time beginning with noon. To reduce to Central Standard time subtract 6 hours, or for Eastern time subtract 5 hours. Alternate minima only are given except in the case of RX Cephei and UZ Cygni.]

U Cephei			Z Persei			RX Cephei			RT Persei			RT Persei		
	d	h		d	h		d	h		d	h		d	h
July	2	15	July	1	2	Aug.	14	0	July	1	21	July	23	23
	7	15		7	5		Algol			3	13		25	16
	12	14		13	7	July	3	21		5	6		27	8
	17	14		19	10		9	15		6	23		29	1
	22	14		25	13		15	8		8	16		30	18
	27	13		31	15		21	2		10	9	Aug.	1	11
Aug.	1	13	Aug.	6	18		26	19		12	1		3	3
	6	13		12	21	Aug.	1	13		13	18		4	20
	11	12		19	0		7	7		15	11		6	13
	16	12		25	2		13	0		17	4		8	6
	21	12		31	5		18	18		18	20		9	22
	26	11	RX Cephei				29	12		20	13		11	15
	31	11	July	12	17		30	5		22	6		12	12

Minima of Variable Stars of the Algol Type.—Continued.

RT Persei			RR Velorum			δ Libræ			Z Herculis			RX Herculis		
	d	h		d	h		d	h		d	h		d	h
Aug.	14	4	July	9	0	Aug.	14	0	July	4	21	Aug.	5	18
	15	21		12	17		18	16		8	21		7	13
	17	14		16	10		23	7		12	21		9	8
	19	7		20	3		27	23		16	20		11	2
	20	23		23	20	U Coronae				20	20		12	21
	22	16		27	13	July	2	11		24	20		14	16
	24	9		31	6		9	9		28	20		16	10
	26	2	Aug.	3	23		16	6	Aug.	1	20		18	5
	27	19		7	16		23	4		5	20		20	0
	29	11		11	9		30	2		9	19		21	18
	31	4		15	2	Aug.	5	23		13	19		23	13
λ Tauri				18	19		12	21		17	19		25	8
July	2	9		22	12		19	19		21	19		27	2
	10	7		26	5		26	17		25	19		28	21
	18	4		29	22	R Arae				29	19		30	16
	26	2	SS Carinae			July	3	1	RS Sagittarii			SX Sagittarii		
Aug.	3	0	July	1	4		11	21	July	3	2	July	2	11
	10	22		7	19		20	17		7	22		6	14
	18	19		14	9		29	14		12	18		10	18
	26	17		20	23	Aug.	7	10		17	14		14	22
RW Tauri				27	14		16	7		22	9		19	2
July	3	12	Aug.	3	4		25	3		27	5		23	5
	9	0		9	19	U Ophiuchi			Aug.	1	1		27	9
	14	14		16	9	July	1	10		5	21		31	13
	20	3		23	0		3	2		10	17	Aug.	4	16
	25	16		29	14		4	18		15	13		8	20
	31	5	Z Draconis				6	10		20	9		13	0
Aug.	5	18	July	1	20		8	3		25	5		17	3
	11	6		4	13		9	19		30	1		21	7
	16	19		7	6		11	11	V Serpentis				25	11
	22	8		9	23		13	4	July	2	5		29	14
	27	21		12	16		14	20		9	2	RR Draconis		
RW Persei				15	9		16	12		16	0	July	2	22
July	4	0		18	3		18	4		22	22		8	14
	30	10		20	20		19	21		29	20		14	6
Aug.	25	19		23	13		21	13	Aug.	5	17		19	22
RS Cephei				26	6		23	5		12	15		25	14
July	5	12		28	23		24	21		19	13		31	6
	30	8		31	16		26	14		26	11	Aug.	5	22
Aug.	24	4	Aug.	3	10		28	6	RX Herculis				11	14
Y Camelopardi				6	3		29	22	July	1	4		17	6
July	3	22		8	20		31	14		2	23		22	21
	10	13		11	13	Aug.	2	7		4	18		28	13
	17	3		14	6		3	23		6	12	RV Lyræ		
	23	18		16	23		5	15		8	7	July	3	0
	30	9		19	16		7	7		10	2		10	5
Aug.	6	0		22	10		9	0		11	21		17	10
	12	14		25	3		10	16		13	15		24	15
	19	5		27	20		12	8		15	10		31	19
	25	20		30	13		14	0		17	5	Aug.	8	0
S Velorum			δ Libræ				15	17		18	23		15	5
July	5	11	July	3	3		17	9		20	18		22	10
	17	8		7	18		19	1		22	13		29	14
	29	5		12	10		20	18		24	7	U Sagittæ		
Aug.	10	2		17	2		22	10		26	2	July	2	6
	21	22		21	17		24	2		27	21		9	0
RR Velorum				26	9		25	18		29	15		15	18
July	1	14		31	1		27	11		31	10		22	13
	5	7	Aug.	4	17		29	3	Aug.	2	5		29	7
				9	8		30	19		3	23			

Minima of Variable Stars of the Algol Type.—Continued.

U Sagittæ

	d	h
Aug.	5	1
	11	19
	18	14
	25	8

SY Cygni

	d	h
July	4	23
	17	0
	29	0
Aug.	10	0
	22	0

WW Cygni

	d	h
July	4	1
	10	17
	17	8
	23	23
	30	14
Aug.	6	6
	12	21
	19	12
	26	3

SW Cygni

	d	h
July	2	11
	11	14
July	20	18
	29	21
Aug.	8	1
	17	4
	26	8

VW Cygni

	d	h
July	7	23
	24	19
Aug.	10	16
	27	13

W Delphini

	d	h
July	3	9
	13	0
	22	15
Aug.	1	5
	10	20
	20	11
	31	1

RR Delphini

	d	h
July	3	10
	12	14
	21	19
	31	0
Aug.	9	5
Aug.	18	10
	27	15

RV Capricorni

	d	h
July	1	15
	4	21
	8	3
	11	9
	14	15
	17	21
	21	3
	24	9
	27	15
	30	22
Aug.	3	4
	6	10
	9	16
	12	22
	16	1
	19	10
	22	16
	25	22
	29	4

VV Cygni

	d	h
July	1	0
	3	23
	6	22
	9	21
	12	20
	15	19
	18	17
	21	16
	24	15
	27	14
	30	13
Aug.	2	12
	5	11
	8	10
	11	9
	14	8
	17	6
	20	5
	23	4
	26	3
	29	2

UZ Cygni

	d	h
July	6	9
Aug.	6	16

RZ Cassiop.

	d	h
July	1	3
July	3	13
	5	22
	8	7
	10	17
	13	2
	15	12
	17	21
	20	6
	22	16
	25	1
	27	10
	29	20
Aug.	1	5
	3	14
	6	0
	8	9
	10	19
	13	4
	15	13
	17	23
	20	8
	22	17
	25	3
	27	12
	29	21

Approximate Magnitudes of Variable Stars on May 1, 1907.

[Communicated by the Director of Harvard College Observatory, Cambridge, Mass.]

Name.	R. A. 1900. h	m	Decl. 1900. °	′	Magn.	Name.	R. A. 1900 h	m	Decl. 1900 °	′	Magn.
T Androm.	0	17.2	+26	26	7.8 *i*	S Camelop.	5	30.2	+68	45	9.2
T Cassiop.		17.8	+55	14	8.5 *i*	RR Tauri		33.3	+26	19	10.0
R Androm.		18.8	+38	1	10.0 *i*	U Aurigae		35.6	+31	59	10.0 *i*
W Cassiop.		49.0	+58	1	9.0 *i*	Z Tauri		46.7	+15	46	8.8 *i*
U Androm.	1	9.8	+40	11	9.5 *i*	U Orionis		49.9	+20	10	6.2 *i*
S Cassiop.		12.3	+72	5	10.5*d*	V Camelop.		49.4	+74	30	14
RU Androm.		32.8	+38	10	9.0 *i*	Z Aurigæ		53.6	+54	18	9.0 *i*
X Cassiop.		49.8	+58	46	10.2*d*	X Aurigae	6	4.4	+50	15	10.0 *i*
U Persei		53.0	+54	20	<7.6 *i*	V Aurigae		16.5	+47	45	9.5*d*
Z Cephei	2	12.8	+81	13	13.8*d*	V Monoc.		17.7	− 2	9	12.5*d*
o Ceti		14.3	− 3	26	6.4*d*	R Monoc.		33.7	+ 8	49	11.8*d*
S Persei		15.7	+58	8	10.0*d*	S Lyncis		35.9	+58	0	13.5
RR Cephei		29.4	+80	42	14.5	W Monoc.		47.5	− 7	2	10.8 *i*
R Trianguli		31.0	+33	50	8.5 *i*	X Monoc.		52.4	− 8	56	8.6 *i*
W Persei		43.2	+56	34	9.0 *i*	R Lyncis		53.0	+55	28	12.6*d*
Y Persei	3	20.9	+43	50	10.8d	RS Gemin.		55.2	+30	40	10.2 *i*
R Persei		23.7	+35	50	9.8*d*	V Can. Min.	7	1.5	+ 9	2	14
Nov. Per. 2		24.4	+43	34	13.0	R Gemin.		1.3	+22	52	12.2 *i*
T Tauri	4	16.2	+19	18	12.4*d*	R Can. Min.		3.2	+10	11	8.2
R Tauri		22.8	+ 9	56	13.8*d*	RR Monoc.		12.4	+1	17	<14
W Tauri		22.2	+15	49	8.8 *i*	V Gemin.		17.6	+13	17	8.8*d*
S Tauri		23.7	+ 9	44	10.0	S Can Min.		27.3	+ 8	32	12.8*d*
T Camelop.		30.4	+65	57	9.3*d*	T Can. Min.		28.4	+11	58	13.8*d*
X Camelop.		32.6	+74	56	8.8 *i*	Z Puppis		28.3	−20	27	8.0
V Tauri		46.2	+17	22	10.0*d*	U Can. Min.		35.9	+ 8	37	10.5*d*
R Orionis		53.6	+ 7	59	12.5*d*	S Gemin.		37.0	+23	41	14.2*d*
R Aurigae	5	9.2	+53	28	13.2 *i*	T Gemin.		43.3	+23	59	8.8
S Aurigae		20.5	+34	4	9.2*d*	U Puppis		56.1	−12	34	14.5*d*
W Aurigae		20.1	+36	49	9.5 *i*	R Cancri	8	11.0	+12	2	9.5 *i*

Approximate Magnitudes of Variable Stars on May 1, 1907—Con.

Name.	R. A. 1900. h	m	Decl. 1900. °	′	Magn.	Name.	R.A. 1900. h	m	Decl. 1900. °	′	Magn.
V Cancri	8	16.0	+17	36	8.7*d*	X Librae	15	30.4	−20	50	11.2
RT Hydræ		24.7	− 5	59	8.3*d*	S Urs. Min.		33.4	+78	58	10.5*d*
U Cancri		30.0	+19	14	14 *d*	R Coronae		44.4	+28	28	5.8
X Urs. Maj.		33.7	+50	30	13	X Coronae		45.2	+36	35	8.0
S Hydrae		48.4	+ 3	27	11.5 *i*	R Serpentis		46.1	+15	26	11.0 *i*
T Hydrae		50.8	− 8	46	8.5*d*	V Coronæ		46.0	+39	52	8.0
T Cancri		51.0	+20	14	10.0	R Librae		47.9	−15	56	9.0
W Cancri	9	4.0	+25	39	13.8*d*	RR Librae		50.6	−18	1	8.5
X Hydrae		30.7	−14	15	11.8*d*	RZ Scorpii		58.6	−23	50	10.8
Y Draconis		31.1	+78	18	13.0	Z Scorpii	16	0.1	−21	28	10.5
R Leo. Min.		39.6	+34	58	11.6*d*	R Herculis		1.7	+18	38	11.2 *i*
RR Hydrae		40.4	−23	34	12.0	U Serpentis		2.5	+10	12	12.0
R Leonis		42.2	+11	54	8.0*d*	X Scorpii		2.7	−21	16	<12
Y Hydrae		46.4	−22	33	8.0	W Scorpii		5.9	−19	53	<11.5
V Leonis		54.5	+21	44	13.5*d*	RX Scorpii		5.9	−24	38	<12
R Urs. Maj.	10	37.6	+69	18	11.6*d*	RU Herculis		6.0	+25	20	10.5
W Leonis		48.4	+14	15	12.8	R Scorpii		11.7	−22	42	11.2*d*
V Hydrae		46.8	−20	43	9.0	S Scorpii		11.7	−22	39	<13.5
S Leonis	11	5.7	+ 6	0	13.6*d*	W Coronae		11.8	+38	3	8.2
R Comae		59.1	+19	20	12.0 *i*	V Ophiuchi		21.2	−12	12	7.5
T Virginis	12	9.5	− 5	29	13.6	U Herculis		21.4	+19	7	11.5 *i*
R Corvi		14.4	−18	42	10.6 *i*	Y Scorpii		23.8	−19	13	10.5
T Can. Ven.		25.2	+32	3	12.2*d*	SS Herculis		28.0	+ 7	3	9.0 *i*
Y Virginis		28.7	− 3	52	12.8*d*	W Herculis		31.7	+37	32	9.6*d*
T Urs. Maj.		31.8	+60	2	7.5 *i*	R Draconis		32.4	+66	58	11.6
R Virginis		33.4	+ 7	32	7.2*d*	S Herculis		47.4	+15	7	9.2*d*
RS Urs. Maj.		34.4	+59	2	10.8 *i*	RV Herculis		56.8	+31	22	13.8
S Urs. Maj.		39.6	+61	38	8.8 *i*	R Ophiuchi	17	2.0	−15	58	8.5
RU Virginis		42.2	+ 4	42	7.8	T Draconis		54.8	+58	14	12.0
U Virginis		46.0	+ 6	6	8.6	T Herculis	18	5.3	+31	0	13.6*d*
RT Virginis		57.6	+ 5	43	8.0	W Draconis		5.4	+65	66	<14
RV Virginis	13	2.7	−12	38	11.5*d*	X Draconis		6.8	+66	8	9.7
R Hydrae		24.2	−22	46	7.6*d*	X Ophiuchi		33.6	+ 8	44	8.2
S Virginis		27.8	+ 6	41	12.5	RY Lyrae		41.2	+34	34	<12
T Urs. Min.		32.6	+73	56	12.6*d*	R Aquilae	19	1.6	+ 8	5	10.0*d*
R Can. Ven.		44.6	+40	2	9.8*d*	T Sagittae		17.2	+17	28	10.0
RR Virginis		59.6	− 8	43	11.6	R Cygni		34.1	+49	58	7.6 *i*
Z Bootis	14	1.7	+13	59	<13	X Cygni		46.7	+32	40	6.8*d*
S Bootis		19.5	+54	16	12.5 *i*	Z Cygni		58.6	+49	46	7.8 *i*
RS Virginis		22.3	+ 5	8	13.5	RS Cygni	20	9.8	+38	28	7.2*d*
V Bootis		25.7	+39	18	7.8	U Cygni		16.5	+47	35	7.8*d*
R Camelop.		25.1	+84	17	9.4 *i*	RW Cygni		25.2	+39	39	8.5
R Bootis		32.8	+27	10	8.3 *i*	V Aquarii		41.8	+ 2	4	8.5
V Librae		34.8	−17	14	13.5	X Cephei	21	3.6	+82	40	11.8*d*
U Bootis		49.7	+18	6	10.0 *i*	T Cephei		8.2	+68	5	6.3 *i*
RT Librae	15	0.8	−18	21	12.8	S Cephei		36.5	+78	10	10.4*d*
T Librae		5.0	−19	38	13.0	S Lacertae		24.6	+39	48	10.0*d*
S Librae		15.6	−20	2	11.8	R Lacertae		38.8	+41	51	<14
S Serpentis		17.0	+14	40	12.2 *i*	V Cassiop.	23	7.4	+59	8	12.5*d*
S Coronae		17.3	+31	44	8.2*d*	Z Cassiop.		39.7	+56	2	9.0*d*
RS Librae		18.5	−22	33	9.6	R Cassiop.		53.3	+50	50	13.2
RU Librae		27.7	−14	59	13.5	Y Cassiop.		58.2	+55	7	10.8*d*

The letter *i* denotes that the light is increasing, the letter *d* that the light is decreasing, the sign <, that the variable is fainter than the appended magnitude.

The magnitudes given above have been compiled by Mr. Leon Campbell of the Harvard College Observatory, from observations made at the Whiteside, Vassar College and Harvard Observatories.

Maxima of Variable Stars of Short Period not of the Algol Type.

Unless otherwise indicated the times of maxima only are given; and the times of minima may be found by subtracting the interval printed in parentheses under the names of the stars.

RW Cassiop.

	d	h
	(−5	19)
July	1	2
	15	22
	30	17
Aug.	14	12
	29	7

RX Aurigæ

	(−4	0)
July	12	3
	23	18
Aug.	4	9
	16	0
	27	15

V Carinæ

	(−2	4)
July	1	9
	8	2
	14	18
	21	11
	28	4
Aug.	3	21
	10	13
	17	6
	23	23
	30	15

T Velorum

	(−1	10)
July	2	22
	7	13
	12	4
	16	20
	21	11
	26	2
	30	18
Aug.	4	9
	9	1
	13	16
	18	7
	22	23
	27	14

W Carinæ

	(−1	0)
July	5	5
	9	14
	13	22
	18	7
	22	16
	27	1
	31	10
Aug.	4	19
	9	4
	13	13
	17	22
	22	7
	26	15
	31	0

S Muscæ

	(−3	11)
July	5	3

S Muscæ

	d	h
July	14	18
	24	10
Aug.	3	2
	12	18
	22	9

T Crucis

	(−2	2)
July	4	14
	11	8
	18	1
	24	19
	31	13
Aug.	7	6
	14	0
	20	17
	27	11

R Crucis

	(−1	10)
July	2	6
	8	2
	13	21
	19	17
	25	13
	31	9
Aug.	6	5
	12	1
	17	20
	23	16
	29	12

S Crucis

	(1−	12)
July	4	3
	8	20
	13	12
	18	5
	22	21
	27	14
Aug.	1	6
	5	23
	10	15
	15	8
	20	1
	24	17
	29	10

W Virginis

	(−8	5)
July	11	16
	28	23
Aug.	15	5

V Centauri

	(−1	11)
July	4	12
	10	0
	15	12
	21	0
	26	11
	31	23
Aug.	6	11
	11	23

V Centauri

	d	h
Aug.	17	11
	22	23
	28	11

R Triang. Austr.

	(−1	0)
July	3	8
	6	17
	10	3
	13	12
	16	21
	20	7
	23	16
	27	1
	30	11
Aug.	2	20
	6	5
	9	15
	13	0
	16	9
	19	19
	23	4
	26	13
	29	23

S Triang. Austr.

	(−2	2)
July	4	7
	10	15
	16	23
	23	7
	29	14
Aug.	4	22
	11	6
	17	14
	23	22
	30	5

S Normæ

	(−4	10)
July	3	4
	12	22
	22	16
Aug.	1	10
	11	4
	20	22
	30	16

RV Scorpii

	(−1	10)
July	1	2
	7	4
	13	5
	19	7
	25	8
	31	10
Aug.	6	11
	12	13
	18	14
	24	16
	30	17

RV Ophiuchi Minimum

	d	h
July	1	20
	5	12
	9	6
	12	21
	16	14
	20	6
	23	23
	27	15
	31	8
Aug.	4	0
	7	17
	11	9
	15	2
	18	18
	22	11
	26	3
	29	20

X Sagittae

	(−2	22)
July	4	13
	11	13
	18	14
	25	14
Aug.	1	14
	8	14
	15	15
	22	15
	29	15

Y Ophiuchi

	(−6	5)
July	11	5
	28	8
Aug.	14	11
	31	14

W Sagittarii

	(−3	0)
July	8	2
	15	16
	23	6
	30	21
Aug.	7	15
	15	5
	22	19
	30	10

Y Sagittarii

	(−2	2)
July	5	6
	11	0
	16	19
	22	13
	28	8
Aug.	3	3
	8	21
	14	16
	20	10
	26	5

U Sagittarii

	d	h
	(−2	23)
July	4	4
	10	22
	17	16
	24	10
	31	4
Aug.	6	22
	13	15
	20	9
	27	3

β Lyrae

	(−3	7)
	(−3	2)
July	7	4
	13	10
	20	2
	26	8
Aug.	2	0
	8	6
	14	22
	21	4
	27	20

κ Pavonis

	(−4	7)
July	9	0
	18	2
	27	4
Aug.	5	7
	14	9
	23	11

U Aquilae

	(−2	4)
July	6	2
	13	3
	20	3
	27	4
Aug.	3	5
	10	5
	17	6
	24	6
	31	7

XZ Cygni Period 11^h

July	1	18
	3	15
	5	12
	7	9
	9	6
	11	2
	12	23
	14	20
	16	17
	18	14
	20	10
	22	7
	24	4
	26	1
	27	21
	29	18
	31	15

Maxima of Variable Stars of the Short Period not of Algol Type.
Continued.

XZ Cygni

	d	h
Aug.	2	12
	4	9
	6	5
	8	2
	9	23
	11	20
	13	17
	15	13
	17	10
	19	7
	21	4
	23	1
	24	21
	26	18
	28	15
	30	12

U Vulpeculae
(−2 3)

	d	h
July	2	9
	10	9
	18	8
	26	8
Aug.	3	7
	11	7
	19	6
	27	6

SU Cygni
(−1 7)

	d	h
July	3	1
	6	4
	10	17
	14	14
	18	10
	22	6
	26	3
	29	23
Aug.	2	19
	6	16
	10	12
	14	8
	18	4
	22	1
	25	21
	29	17

η Aquilae
(−2 6)

	d	h
July	5	9

η Aquilae

	d	h
July	12	13
	19	17
	26	21
Aug.	3	1
	10	6
	17	10
	24	14
	31	18

S Sagittae
(−3 10)

	d	h
July	3	15
	12	1
	20	10
	28	19
Aug.	6	4
	14	13
	31	8

V Vulpeculae
Minimum

	d	h
July	14	20
Aug.	21	15

X Cygni
(−6 19)

	d	h
July	4	0
	20	9
Aug.	5	18
	22	4

T Vulpeculae
(−1 10)

	d	h
July	1	0
	5	11
	9	21
	14	7
	18	18
	23	4
	27	15
Aug.	1	1
	5	12
	9	22
	14	9
	18	19
	23	6
	27	16

TX Cygni

	d	h
July	7	12
	22	5
Aug.	5	23
	20	16

WZ Cygni
Minimum
Period 14h.

	d	h
July	1	8
	3	16
	6	0
	8	8
	10	17
	13	1
	15	9
	17	17
	20	1
	22	9
	24	17
	27	1
	29	9
	31	18
Aug.	3	2
	2	10
	7	18
	10	2
	12	10
	14	18
	17	2
	19	10
	21	19
	24	3
	26	11
	28	19
	31	3

RV Capricorni
Period 10h.7

	d	h
July	1	6
	3	1
	4	20
	6	15
	8	10
	10	5
	12	0
	13	19
	15	14
	17	9
	19	4
	20	23
	22	18
	24	13
	26	7
	28	2

RV Capricorni

	d	h
July	29	21
	31	16
Aug.	2	11
	4	6
	6	1
	7	20
	9	15
	11	10
	13	5
	15	0
	16	19
	18	14
	20	9
	22	4
	23	23
	25	18
	27	13
	29	8
	31	3

VY Cygni
(−2 2)

	d	h
July	4	14
	12	10
	20	7
	28	3
Aug.	5	0
	12	20
	20	17
	28	14

VZ Cygni
(−2 12)
(−3 6)

	d	h
July	3	11
	8	11
	13	4
	18	4
	22	21
	27	21
Aug.	1	15
	6	15
	11	8
	16	8
	21	2
	26	2
	30	19

δ Cephei
(−1 10)

	d	h
July	3	3
	8	12
	13	21
	19	6
	24	15
	29	23
Aug.	4	8
	9	17
	15	2
	20	11
	25	19
	31	4

V Lacertæ
(−1 17)

	d	h
July	1	3
	6	3
	11	2
	16	2
	21	1
	26	1
	31	1
Aug.	5	0
	10	0
	14	23
	19	23
	24	23
	29	22

88.1906 Lacertæ
Minimum

	d	h
July	3	23
	9	10
	14	20
	20	7
	25	17
	31	4
Aug.	5	14
	11	1
	16	12
	21	22
	27	8

RV Persei.—In *A.N.* 4173 Mr. Sigurd Enebo of Dombaas, Norway, gives elements of this variable depending upon a series of observations by himself extending from December 16, 1905 to February 7, 1907. He observed minima on the dates November 9, and 21, 1906 and January 31, February 2, and 4, 1907. These give the approximate elements:

Minimum = 1906 Nov. $21^d\ 7^h$ Gr. M. T. + $1^d\ 23^h\ 17^m$ E.
= J. D. 2417536.29 + 1.970 E.

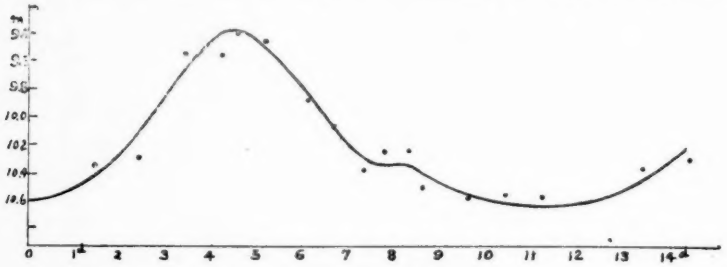
Mr. Enebo thinks that this variable is of the Algol Type, the constant phase lasting 41 hours and the light change 7 hours. Later minima were observed

March 31, $11^h\ 30^m$ and April 2, $11^h\ 0^m$ Gr. M. T., which confirm the above elements. The position of RV Persei for 1900 is

R. A. $4^h\ 04^m\ 09^s$ Decl. $+\ 33°\ 59'.4$

and the range of brightness from 9.5 to <12 magnitude.

RY Cassiopeiæ (28.1906.) In *A. J.* No. 593 Mr. Naozo Ichinohe gives the light curve of this variable, which appears to be of the δ Cephei type. "The light increases pretty rapidly after it passes the minimum, and reaches its maximum in five days, and then it begins to decrease, rapidly at first, but slower than the rise. At seven and one-half days after minimum its magnitude becomes



Light Curve of RY Cassiopeiæ

about $10^m.4$, and then the decrease of light stops a little while; or becomes very slow and the star remains at the minimum about three days." The provisional period is 12.07 days, and the formula for computing maxima:

Maximum = 1906 Nov. 219 Gr. M. T. $+\ 12^d.07$ E

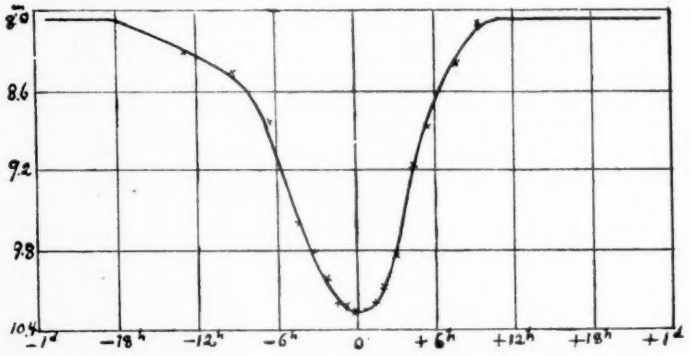
$=$ J. D. 2417536.9 $+$ 12.07 E

The position of the star for 1900 is

R. A. $23^h\ 47^m\ 10^s$ Decl. $+\ 58°\ 11'.2$

and the magnitude at maximum is about 9.4.

The Variable 120.1906 Persei.—In *A. N.* 4172 Mr. Naozo Ichinohe of the Yerkes Observatory gives new elements of this variable depending upon observations November 8, 10, 13, 15, 18, 22, 23, 24, 27, 28, December 1, 8, 19,



Light Curve of 120.1906 Persei

1906, and January 8, 22, February 5, 19, 22 and 26, 1907. Minima were observed completely on December 19 and February 5. On February 22 the star was at its normal brightness, so that the period of 3.43 days is precluded. The new elements are

Minimum = 1906 Nov. 8, $12^h\ 23^m$ Gr. M. T. + $6^d\ 20^h\ 41^m.4$ E
= J. D. 2417523.516 + $6^d.862$ E.

Mr. Ichinohe gives the normal brightness of the star as $8^m.0$ and the minimum as $10^m.3$.

Algol Variable 24.1907 Monocerotis.—In *A. N.* 4174 Mr. Naozo Ichinohe gives the following preliminary elements of this variable:—

Minimum = $2417680^d.653$ Gr. M. T. + $1.^d90686$ E
= 1907 Apr. 14 $15^h\ 40^m$ Gr. M. T. + $1^d\ 21^h\ 43^m\ 59^s.5$ E.

He observed the greater part of the decreasing, as well as some of the increasing phase of the variation on the night of April 14, and again a portion of the change on April 16, but could not follow it through the minimum on the latter night. Fractions of the period $1^d.90$ are excluded by observations on ten other nights in March and April when the star's light was normal. The range of variation is about $9^m.0 - 10^m.8$ and the duration of the eclipse 6 or 7 hours. The following ephemeris has been computed from the elements given above:

MINIMA OF 27.1997 MONOCEROTIS.

	h		h		h		h
July 1	4	July 16	11	Aug. 2	14	Aug. 17	20
3	2	18	8	4	12	19	18
5	0	20	6	6	10	21	16
6	22	22	4	8	8	23	14
8	19	24	2	10	5	25	12
10	17	25	23	12	3	27	9
12	16	27	21	14	1	29	7
14	14	29	18	15	23	31	5
		31	17				

Three New Variables.—In *A. N.* 4170 Professor W. Ceraski announces three new variables discovered by Mme. L. Ceraski upon the Moscow photographs. Their positions are as follows:

Designation	R.A. 1855 h	m	s	Decl. 1855 °	′	R.A. 1900 h	m	s	Decl. 1900 °	′
26.1907 Draconis	18	21	10	+58	48.6	18	21	49	+58	50.1
27.1907 Aurigæ	5	08	28	+38	09.8	5	11	32	+38	13.1
28.1907 Ursæ Majoris	9	01	36	+67	51.3	9	05	37	+67	40.5

The first (26.1907) varies between 9.5 and 10.2 magnitude and the period is probably less than 24 hours. The second (27.1907) is possibly of the Algol type and varies from 10.5 to about 12 magnitude. The period is undetermined. The third (28.1907) varies from 11.0 to 11.7 magnitude, but the period is unknown. This star is + 67° No. 3211 of the Greenwich Astrographic Catalogue.

Variable Star Notes.

o CETI

As shown in the light curve in POPULAR ASTRONOMY for last February, page 96, *o* Ceti had decreased in light one-fifth of a magnitude on January the third 1907; and though clouds and low altitude prevented observations of its decline to minimum, the following few estimates obtained indicate the usual slow change.

1907 January 8, 14, 19. It equals α Ceti of 2.6 magnitude.
January 20, 21. Less than α Ceti in light of half Moon.
February 6, 11. To the naked eye it is equal to Delta Ceti of 4 magnitude but in opera glass seems brighter. Night clear and moonless.
February 27, With opera glass less than Delta Ceti.
March 3, 9 with opera glass, brighter than 69 Ceti of 5.5 magnitude.

The corrected date of maximum, namely December 19, 1906, given in *Ast. Jour.* No. 560 was included in the weeks of greatest lustre.

R LEONIS

The magnitudes of the comparison stars, from which the following estimates were deduced, are those of the D.M. or *Connaissance Des Temps*.

1906 January	29	Of 8.1 magnitude.	
February	11, 18,	" 7.1	"
February	25	6.9	"
March	2, 5,	6.3	"
March	12	5.9	"
March	16, 18	5.7	"
March	26	5.8	"
March	31	5.7	"
April	6	5.7	" Moonlight
April	10	5.8	"
April	14	5.9	"
April	27	6.4	"
May	5	6.7	" Moonlight
May	11, 17,	7.1	"
May	29	7.3	"
1907 February	6	5.7	"
February	11, 12,	5.6	"
February	27	5.9	" Full Moon
March	4, 7.	5.9	" In telescope, but in opera-glass 6.3.
March	29	6.7	" In telescope.
April	8	7.	"

The maximum was predicted for March 12.

U GEMINORUM

1906 November 19, $0^h: 15^m$. A. M. Of 9.6 magnitude. Night very clear.
November 20, 10:35. P. M. Nearly equal to an adjacent star of 9.3 magnitude. Night slightly hazy.
November 21, 11:40. P. M. Decreased but distinctly brighter than stars of 10.3 magnitude. Altitude high. Night clear.
November 23, 10:25. P. M. The same. Night clear. Faint moonlight.
November 24, 11. P. M. The same. The variable is not sharply defined.
November 25, 11:15 P. M. Fainter but still brighter than 10.3 magnitude. Definition is not as sharp for U as for the adjacent stars. Gibbous Moon.
November 26, 10:30 P. M. Much fainter than the adjacent star of 9.3 magnitude. Those less than tenth are not discernible in the light of the gibbous Moon.
November 27, 11:46. Not discernible in bright moonlight.

This maximum seems to have been rather brief if the rise to 9.6 magnitude was as rapid as usual. The comparison stars used are those in POPULAR ASTRONOMY, May 1894, page 400.

ROSE O'HALLORAN.

San Francisco, April 9, 1907.

GENERAL NOTES.

Professor W. H. Pickering is still at work on the orbit of the tenth satellite of Saturn. The elements of the orbit of this satellite were published in Vol. 53 of the *Harvard Annals*. These elements were based on the negatives taken at Arequipa in 1904. In comparing these results with those found from plates taken in 1900, large difference is noticed. The orbit of 1904 was very eccentric, while the orbit of 1900 was nearly circular. It was also noticed that the new satellite had passed very near to Titan, and Professor Pickering suggests that these remarkable changes in its orbit might be due to disturbances caused by that greatest satellite in the Saturnian system. These facts, if rightly stated will give some interesting problems to the student of celestial mechanics.

U Geminorum. For a complete discussion of the light change of U Geminorum, readers of this paper will please me very much by sending me their complete observations of this remarkable star, or to tell me where I can find some unpublished observations. Any observations however few they may be, are welcome.

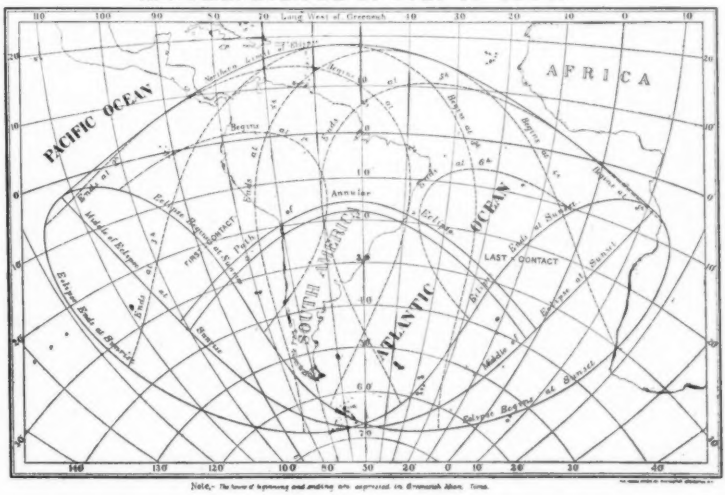
Mr. J. A. Parkhurst, Mr. Paul S. Yendell, Mr. William E. Sperra and Mr. Zaccheus Daniel have been so kind as to send me theirs and others will be appreciated.

Delft, Holland.
April 10, 1907.

J. Voûte.

An Annular Eclipse of the Sun July 10, 1907.—This will not be visible in North America, but will be visible as a partial eclipse over the whole of South America and adjacent parts of the Atlantic and Pacific Oceans. Just at

ANNULAR ECLIPSE OF JULY 10TH 1907.



sunset the beginning may be observed on the west coast of South Africa. The path of annular eclipse crosses Bolivia and Brazil. In Bolivia the eclipse begins

between eight and nine o'clock in the morning and ends between noon and one o'clock in the afternoon. In Brazil the phases are about an hour later.

A Partial Eclipse of the Moon, July 24, 1907.—This will be visible in the United States and generally throughout North and South America, except in the northwestern part of North America. It will be partly visible in Europe and Africa.

TIMES OF THE PHASES

	Greenwich M.T.			Central Standard Time.	
		h	m	h	m
Moon enters penumbra	July 24	13	58.7	7	58.7
Moon enters shadow		15	03.7	9	03.7
Middle of the Eclipse		16	22.4	10	22.4
Moon leaves shadow		17	41.1	11	41.1
Moon leaves penumbra		18	46.2	12	46.2

Magnitude of eclipse = 0.620 (Moon's diameter = 1)

The Friendly Stars. A book unique in the annals of the astronomical world and one which will prove very useful to teachers has just been published. It is entitled "The Friendly Stars" and is what its name implies, a book which has grown out of the personal observations of the writer who has for years greeted the nightly appearance of the stars as a gathering of friends. In untechnical but scientifically accurate language, the author, Mrs. Martha Evans Martin, shows how and when and where to look for the brightest stars, the choicest individuals, as she calls them, among the heavenly host. She has collected all that is individually important about them and placed these leading facts intelligently under each star by name. For example she tells us that Arcturus rises for easy observation in February "pointed out by the handle of the dipper. April is its month of greatest glory, yet it graces the evening skies until December when the pointing handle of the dipper shows us that it disappears almost with the setting Sun." To this she adds the latest information about its color, size, distance, type, motion, heat and other cold, scientific facts, which are not cold under her pen but are clothed with the gracious charm of the trained writer who has also eyes to see "the day's progress in beauty" from the tender bloom of the cheerful hepatica to the gay song of the lilting lark and the grand glory of blazing Arcturus.

The book has chapters on the number, names, light and distances of the stars, all written with the same charm and clearness and closes with diagrams and directions for tracing the principal constellations. It is a book to be commended to those who know nothing of astronomy, for it will arouse within them a desire to know something of the wonderful beauty of the universe; it is to be commended to teachers, for it contains facts which have been gathered from many sources but which are here collected in a form for easy reference; it is to be commended to public libraries, for its language is popular and its facts are reliable.

The book is typographically pleasing, being published by Harper and Brothers and it has an interesting note by Professor Harold Jacoby, Rutherfurd Professor of Astronomy, Columbia University.

F. E. H.

In the May number of the *Bulletin* of the *Astronomical Society of France*, is an interesting article on the progress of Astronomy by C.-Ed Casparl, president of the Society. It is the leader for that month. Louis Millard's article on the measure of altitudes by the barometer is noteworthy because of the study of formulæ adapted to this kind of work.

Recent Measurements of the Distance of the Sun. In 1905 A. R. Hinks of England delivered a lecture at the Royal Engineer's Institute on the recent measurements of the distance of the Sun. This important address was reprinted in the Smithsonian Report for 1905 pp. 101-118. Another reprint copy from the last source named bearing date 1907 has just reached us. Although like light coming from half the distance of the nearest fixed star it is still interesting and useful to the readers of this Journal. Mr. Hinks has put the matter then at hand very plainly, directly and impressively. He shows how the geodecist and the astronomer must work together in order to solve this great physical problem. If geodesy will furnish the exact size of the Earth, then astronomy will find the exact distance of the Sun in miles.

The outstanding error in astronomical units is about 0″.01, but that means 106,000 miles.

While the Eros campaign was in progress, from 1900 and later, more than fifty observatories in the northern hemisphere of the globe were engaged in work on that planet, for the sole purpose of finding the distance of the Sun from the Earth more accurately than it had been previously known.

From 1801 to 1867, the parallax was changed from 9″.0 to 8″.848. In less than a score of years following, the value for the parallax of the Sun was again reduced to about 8″.80. The methods that contribute to this result were:—

Gill's heliometer, minor planets	8″.802
Constant of aberration of light	8 .799
Parallactic inequality of Moon	8 .794
Mass of Earth from motion of node of Venus	8. 762
Mass of Earth from secular variation of four inner planets	8. 759

The latest and the newest method which seriously began was in 1900 by the photographic study of the planet Eros. The details of this work have been recently and fully published in many scientific journals. In this address the important matter coming out of the Eros campaign is given in very useful way.

Absolute Wave Lengths from Gratings. In the May number of the *Astrophysical Journal*, Ernest Geising has contributed the leading article on absolute wave lengths from gratings. That which appears in the *Journal* is an abstract of a fuller paper by the author published in 1906. The paper, as far as read carefully, is mathematically strong.

In one paragraph is an interesting opinion expressed. It is said that in Mr. Schönroch's opinion the plane surfaces by Brashear are quite as perfect, if not more so, than the quartz surfaces by Zeiss.

The Kinetic Theory of Gases. Elsewhere in this issue will be found an article relating to some serious errors in the kinetic theory of gases, and in the application to it of Newton's second law of motion.

This valuable paper is by Luigi d'Auria, and is based on a new theorem contributed by the author, which he states in the following language:—

The kinetic energy of a smooth, hard and perfectly elastic sphere which is forced to oscillate between two perfectly elastic parallel plates perpendicular to the direction of its motion, while one of these plates is moving towards the other with uniform speed which is very small compared with that of the sphere, varies inversely with the length of the path described by the sphere while oscillating between the plates.

By the use of this theorem, as will be seen the author finds some surprisingly

wonderful results. If he is right in his mathematics, (and we think he is) it is difficult to find in the whole range of science a parallel to this case. "Here we have a theory, which in the words of a great physicist, was shadowed forth by Lucretius, stated by Daniel Bernnoulli, largely developed by Herapath, made a reality by Joule, worked out to its present advanced state by Clausius and Maxwell and declared to be the greatest achievement yet made in molecular science. How could any one imagine that the results of such a theory, in its present advanced state, could be vitiated by such errors as have been pointed out in this article involving as they do a wrong application of Newton's second law of motion?"

In view of the mathematics here given, and all the history of this investigation which must be known to the scholarly physicist, it is a wonder even now that the author has had the patience, perseverance and the courage to surmount all the difficulties in his way.

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Back volumes or single numbers are still on sale at the old prices, $2.50 and 30 cents, respectively.

WM. W. PAYNE,
Northfield, Minn., U. S. A.